1.4 Диференцирање сложене и имплицитне функције

N

1. $z = xz^2 + y \cdot e^z + 2 \qquad \frac{\partial z}{\partial x} = ?$

$z_x = z^2 + x \, 2z \cdot z_x + y \cdot e^z \cdot z_x$

$z_x - 2xz \cdot z_x - y \cdot e^z \cdot z_x = z^2$

$z_x (1 - 2xz - y \cdot e^z) = z^2$

$$\boxed{z_x = \frac{z^2}{1 - 2xz - ye^z}}$$

2. $xyz + \frac{xy}{z} = 1 \qquad x\frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = ? \qquad x \cdot z_x + y \cdot z_y = ?$

$xyz + \frac{xy}{z} = 1 \ \Big/ \frac{\partial}{\partial x}$

$yz + xy \cdot z_x + \frac{yz - xy \cdot z_x}{z^2} = 0$

$yz^3 + xyz^2 \cdot z_x + yz - xy \cdot z_x = 0$

$yz^3 + yz = z_x (xy - xyz^2)$

$z_x = \frac{yz(z^2+1)}{xy(1-z^2)}$

$$\boxed{z_x = \frac{z(1+z^2)}{x(1-z^2)}}$$

$xyz + \frac{xy}{z} = 1 \ \Big/ \frac{\partial}{\partial y}$

$x \cdot z + xy \cdot z_y + \frac{xz - xyz_y}{z^2} = 0$

$xz^3 + xyz^2 z_y + xz - xyz_y = 0$

$xz^3 + xz = z_y (xy - xyz^2)$

$z_y = \frac{xz(z^2+1)}{xy(1-z^2)}$

$$\boxed{z_y = \frac{z(1+z^2)}{y(1-z^2)}}$$

$A = x\frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y}$

$A = \cancel{x} \cdot \frac{z(1+z^2)}{\cancel{x}(1-z^2)} + \cancel{y} \frac{z(1+z^2)}{\cancel{y}(1-z^2)}$

$$\boxed{A = \frac{2z(1+z^2)}{1-z^2}}$$

3. $z = x\cos z - z\sin y \qquad \frac{\partial z}{\partial x} = ?$

$z_x = \cos z - x\sin z \cdot z_x - z_x \sin y$

$z_x + z_x x\sin z + z_x \sin y = \cos z$

$z_x (1 + x\sin z + \sin y) = \cos z$

$$\boxed{z_x = \frac{\cos z}{1 + x\sin z + \sin y}}$$

4. $z = z(x,y)$

$x + y + z = e^z$

$dz = ?$

$x + y + z = e^z \quad / \frac{\partial}{\partial x}$

$1 + z_x = e^z \cdot z_x$

$z_x - e^z \cdot z_x = -1$

$z_x(1 - e^z) = -1$

$z_x = \frac{1}{e^z - 1}$

$x + y + z = e^z \quad / \frac{\partial}{\partial y}$

$1 + z_y = e^z \cdot z_y$

$z_y - e^z z_y = -1$

$z_y(1 - e^z) = -1$

$z_y = \frac{1}{e^z - 1}$

$$\boxed{dz = \frac{dx}{e^z - 1} + \frac{dy}{e^z - 1}}$$

5. $z = z(x,y)$

$x^3 + y^3 + z^3 = 3xyz$

$dz = ?$

$x^3 + y^3 + z^3 = 3xyz \quad / \frac{\partial}{\partial x}$

$\cancel{3}x^2 + \cancel{3}z^2 \cdot z_x = \cancel{3}yz + \cancel{3}xy \cdot z_x$

$z^2 \cdot z_x - xy \cdot z_x = yz - x^2$

$z_x(z^2 - xy) = yz - x^2$

$z_x = \frac{yz - x^2}{z^2 - xy}$

$x^3 + y^3 + z^3 = 3xyz \quad / \frac{\partial}{\partial y}$

$\cancel{3}y^2 + \cancel{3}z^2 \cdot z_y = \cancel{3}xz + \cancel{3}xy\, z_y$

$z^2 \cdot z_y - xy \cdot z_y = xz - y^2$

$z_y(z^2 - xy) = xz - y^2$

$z_y = \frac{xz - y^2}{z^2 - xy}$

$$\boxed{dz = \frac{yz - x^2}{z^2 - xy}\,dx + \frac{xz - y^2}{z^2 - xy}\,dy}$$

6. $z = z(x,y)$; $x = z \ln\left(\frac{z}{y}\right)$; $dz = ?$

$x = z \ln\left(\frac{z}{y}\right) \quad / \frac{\partial}{\partial x}$

$1 = z_x \cdot \ln\left(\frac{z}{y}\right) + \cancel{z} \cdot \frac{1}{\left(\frac{\cancel{z}}{y}\right)} \cdot \frac{z_x}{\cancel{y}}$

$1 = z_x \cdot \ln\left(\frac{z}{y}\right) + z_x$

$1 = z_x\left(\ln\left(\frac{z}{y}\right) + 1\right)$

$z_x = \frac{1}{\ln\left(\frac{z}{y}\right) + 1}$

$x = z \ln\left(\frac{z}{y}\right) \quad / \frac{\partial}{\partial y}$

$0 = z_y \cdot \ln\left(\frac{z}{y}\right) + z \cdot \frac{1}{\left(\frac{z}{y}\right)} \cdot \frac{z_y\, y - z}{y^2}$

$z_y \ln\left(\frac{z}{y}\right) + \frac{z_y \cdot y - z}{y} = 0$

$z_y \cdot y \cdot \ln\left(\frac{z}{y}\right) + z_y \cdot y - z = 0$

$z_y\left(y \ln\left(\frac{z}{y}\right) + y\right) = z$

$z_y = \frac{z}{y \ln\left(\frac{z}{y}\right) + y}$

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$$\boxed{dz = \frac{dx}{\ln\left(\frac{z}{y}\right)+1} + \frac{z\,dy}{y\ln\left(\frac{z}{y}\right)+y}}$$

(7) $z = z(x,y)$; $xy + yz + zx = 1$; $dz = ?$

$xy + yz + zx = 1 \;/\; \frac{\partial}{\partial x}$ $\qquad$ $xy + yz + zx = 1 \;/\; \frac{\partial}{\partial y}$

$y + y\cdot z_x + z_x \cdot x + z = 0$ $\qquad$ $x + z + y\cdot z_y + z_y \cdot x = 0$

$z_x(y+x) = -y - z$ $\qquad$ $z_y(y+x) = -x - z$

$z_x = \frac{-y-z}{y+x}$ $\qquad$ $z_y = \frac{-x-z}{y+x}$

$$\boxed{dz = \frac{-y-z}{y+x}\,dx + \frac{-x-z}{y+x}\,dy}$$

(8) $z = z(x,y)$; $x^3 + y^3 + z^3 = 3xyz$, $dz = ?$ $\qquad$ (8.) = (5.)

(9) $z = z(x,y)$; $x\cdot e^y + y\cdot e^x + z\cdot e^x = 1$; $z_x + z_y = ?$

$xe^y + ye^x + ze^x = 1 \;/\; \frac{\partial}{\partial x}$ $\qquad$ $xe^y + ye^x + ze^x = 1 \;/\; \frac{\partial}{\partial y}$

$e^y + y\cdot e^x + z_x\cdot e^x + z\cdot e^x = 0$ $\qquad$ $xe^y + e^x + z_y\cdot e^x = 0$

$z_x \cdot e^x = -e^y - ye^x - ze^x$ $\qquad$ $z_y\cdot e^x = -xe^y - e^x$

$\underline{z_x = -e^{y-x} - y - z}$ $\qquad$ $\underline{z_y = -x\cdot e^{y-x} - 1}$

$$\boxed{z_x + z_y = -e^{y-x} - y - z - xe^{y-x} - 1}$$

(10) $z = z(x,y)$; $z - x = \operatorname{arctg}\frac{y}{z-x}$; $z_x + z_y = ?$

$z - x = \operatorname{arctg}\frac{y}{z-x} \;/\; \frac{\partial}{\partial x}$

$z_x - 1 = \frac{1}{1+\frac{y^2}{(z-x)^2}}\cdot\left(-\frac{y}{(z-x)^2}\right)\cdot(z_x - 1)$

$z_x - 1 = -\frac{\cancel{(z-x)^2}}{(z-x)^2+y^2}\cdot\frac{y(z_x-1)}{\cancel{(z-x)^2}}$

$z_x - 1 = -\frac{y\cdot(z_x-1)}{(z-x)^2+y^2}$

$y(1-z_x) = (z_x-1)\left((z-x)^2+y^2\right)$

$y - z_x y = z_x\left((z-x)^2+y^2\right) - (z-x)^2 - y^2$

$z_x\left((z-x)^2+y^2\right) + z_x\cdot y = y + (z-x)^2 + y^2 \Rightarrow$ $\quad z_x\left((z-x)^2+y^2+y\right) = (z-x)^2+y^2+y$

$\boxed{z_x = 1}$

$$z-x = \operatorname{arctg} \frac{y}{z-x} \quad \Big/ \frac{\partial}{\partial y}$$

$$z_y = \frac{1}{1+\frac{y^2}{(z-x)^2}} \cdot \frac{z-x-y\cdot z_y}{(z-x)^2}$$

$$z_y = \frac{\cancel{(z-x)^2}}{(z-x)^2+y^2} \cdot \frac{z-x-y\cdot z_y}{\cancel{(z-x)^2}}$$

$$z_y = \frac{z-x-y\cdot z_y}{(z-x)^2+y^2}$$

$$z-x-y\cdot z_y = z_y((z-x)^2+y^2)$$

$$z_y((z-x)^2+y^2) + z_y \cdot y = z-x$$

$$z_y((z-x)^2+y^2+y) = z-x$$

$$z_y = \frac{z-x}{(z-x)^2+y^2+y}$$

$$\boxed{z_x+z_y = 1+\frac{z-x}{(z-x)^2+y^2+y}}$$

(11) $z=z(x,y)$; $xz^5+y^3z = x^3$; $z(1,0)=1$; $d^2z(1,0)$

$$xz^5+y^2z = x^3 \quad \Big/ \frac{\partial}{\partial x}$$

$$z^5+x\cdot 5z^4\cdot z_x + y^3 z_x = 3x^2$$

$$z_x(x\cdot 5z^4+y^3) = 3x^2-z^5$$

$$z_x = \frac{3x^2-z^5}{x\cdot 5z^4+y^3} = \frac{3-1}{5} = \frac{2}{5}$$

$$xz^5+y^3z = x^3 \quad \Big/ \frac{\partial}{\partial y}$$

$$x\cdot 5z^4\cdot z_y + 3y^2z + y^3\cdot z_y = 0$$

$$z_y(x\cdot 5z^4+y^3) = -3y^2z$$

$$z_y = -\frac{3y^2z}{x5z^4+y^3} = 0$$

$$z_{xx} = \frac{(6x-5z^4\cdot z_x)(x5z^4+y^3)-(3x^2-z^5)(5z^4+x\cdot 20z^3\cdot z_x)}{(x\cdot 5z^4+y^3)^2}$$

$$z_{xx} = \frac{(6-5\cdot\frac{2}{5})(5+0)-(3-1)(5+20\cdot\frac{2}{5})}{(5+0)^2} = \frac{4\cdot 5-2\cdot 13}{25} = \frac{20-26}{25} = -\frac{6}{25}$$

$$z_{xy} = \frac{-5z^4\cdot z_y\cdot(x5z^4+y^3)-(3x^2-z^5)\cdot(x\cdot 20z^3\cdot z_y+3y^2)}{(x\cdot 5z^4+y^3)^2}$$

$$z_{xy} = \frac{-(3-1)\cdot 0}{5^2} = 0$$

$$z_{yy} = \frac{(-6yz-3y^2z_x)(x5z^4+y^3)+3y^2z(x\cdot 20z^3 z_x+3y^2)}{(x5z^4+y^3)^2}$$

$$z_{yy} = 0$$

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$$\boxed{d^2z(1,0) = -\frac{6}{25}d^2x}$$

(12.) $z = z(x,y)$; $x - y + 3z + \sin(y-z) = 2$

$$\frac{\partial^2 z}{\partial x^2}\cdot\frac{\partial^2 z}{\partial y^2} - \left(\frac{\partial^2 z}{\partial x \partial y}\right)^2 = ?$$

$$z_{xx}\cdot z_{yy} - (z_{xy})^2 = A$$

$$x - y + 3z + \sin(y-z) = 2 \ \Big/ \frac{\partial}{\partial x}$$

$$1 + 3z_x + \cos(y-z)(-z_x) = 0$$

$$z_x(3-\cos(y-z)) = -1$$

$$\boxed{z_x = \frac{-1}{3-\cos(y-z)}}$$

$$x - y + 3z + \sin(y-z) = 2 \ \Big/ \frac{\partial}{\partial y}$$

$$-1 + 3z_y + \cos(y-z)\cdot(1-z_y) = 0$$

$$-1 + 3z_y - z_y\cos(y-z) + \cos(y-z) = 0$$

$$z_y(3-\cos(y-z)) = 1-\cos(y-z)$$

$$\boxed{z_y = \frac{1-\cos(y-z)}{3-\cos(y-z)}}$$

$$z_{xx} = \frac{+\sin(y-z)\cdot(-z_x)}{(3-\cos(y-z))^2} = \frac{-\sin(y-z)\cdot z_x}{(\cos(y-z)-3)^2} = \boxed{\frac{\sin(y-z)}{(3-\cos(y-z))^3}}$$

$$z_{xy} = -\frac{\sin(y-z)(1-z_y)}{(\cos(y-z)-3)^2} = \frac{\sin(y-z)\cdot(1-z_y)}{(\cos(y-z)-3)^2} = \frac{\sin(y-z) - z_y(\sin(y-z))}{(\cos(y-z)-3)^2} =$$

$$= \frac{\sin(y-z)(3-\cos(y-z)) - (1-\cos(y-z))\sin(y-z)}{(3-\cos(y-z))(\cos(y-z)-3)^2} = \boxed{\frac{2\sin(y-z)}{(3-\cos(y-z))^3}}$$

$$z_{yy} = \frac{[\sin(y-z)\cdot(1-z_y)](3-\cos(y-z)) - (1-\cos(y-z))\cdot\sin(y-z)(1-z_y)}{(3-\cos(y-z))^2} =$$

$$= \frac{\sin(y-z)\cdot(1-z_y)\left(3-\cancel{\cos(y-z)}-1+\cancel{\cos(y-z)}\right)}{(3-\cos(y-z))^2} =$$

$$= \frac{(2-2z_y)\cdot\sin(y-z)}{(3-\cos(y-z))^2} = \frac{2\sin(y-z) - 2z_y\sin(y-z)}{(3-\cos(y-z))^2} =$$

$$= \frac{2\sin(y-z)(3-\cos(y-z)) - 2(1-\cos(y-z))\sin(y-z)}{(3-\cos(y-z))^3} =$$

$$= \frac{2\sin(y-z)\left(3-\cancel{\cos(y-z)}-1+\cancel{\cos(y-z)}\right)}{(3-\cos(y-z))^3} = \boxed{\frac{4\sin(y-z)}{(3-\cos(y-z))^3}}$$

$$A = \frac{\sin(y-z)}{(3-\cos(y-z))^3}\cdot\frac{4\sin(y-z)}{(3-\cos(y-z))^3} - \frac{4\sin^2(y-z)}{(3-\cos(y-z))^6} = 0$$

$$\boxed{A = 0}$$

(13.) $z = z(x,y)$; $z(x,y) = f(xy) + g\left(\frac{x}{y}\right)$

$x^2 \cdot z_{xx} - y^2 \cdot z_{yy} + x \cdot z_x - y \cdot z_y = ?$

$u = xy$ $\qquad M = \frac{x}{y}$

$u_x = y$ $\qquad M_x = \frac{1}{y}$

$u_y = x$ $\qquad M_y = -\frac{x}{y^2}$

$z(x,y) = f(u) + g(M)$

$z_x = z_u \cdot u_x + z_M \cdot M_x = \boxed{z_u \cdot y + z_M \cdot \frac{1}{y}}$

$z_{xx} = (z_x)'_x = \left(z_u \cdot y + z_M \cdot \frac{1}{y}\right)'_x =$

$= (z_{uu} \cdot u_x + z_{uM} \cdot M_x)\, y + (z_{MM} M_x + z_{Mu} u_x) \cdot \frac{1}{y} =$

$= \left(z_{uu} \cdot y + z_{uM} \cdot \frac{1}{y}\right) y + \left(z_{MM} \cdot \frac{1}{y} + z_{Mu} \cdot y\right) \frac{1}{y} =$

$= z_{uu} \cdot y^2 + z_{uM} + z_{MM} \cdot \frac{1}{y^2} + z_{uM} =$

$= \boxed{y^2 \cdot z_{uu} + 2 z_{uM} + \frac{1}{y^2} \cdot z_{MM}}$

$z_y = z_u \cdot u_y + z_M \cdot M_y = \boxed{z_u \cdot x - z_M \cdot \frac{x}{y^2}}$

$z_{yy} = (z_{uu} \cdot u_y + z_{uM} M_y)\, x - (z_{MM} M_y + z_{Mu} u_y) \frac{x}{y^2} =$

$= \left(z_{uu} \cdot x + z_{uM} \cdot \left(-\frac{x}{y^2}\right)\right) x - \left(z_{MM} \cdot \left(-\frac{x}{y^2}\right) + z_{Mu} \cdot x\right) \cdot \frac{x}{y^2} =$

$= z_{uu} \cdot x^2 - z_{uM} \cdot \frac{x^2}{y^2} + z_{MM} \cdot \frac{x^2}{y^4} - z_{Mu} \cdot \frac{x^2}{y^2} =$

$= \boxed{x^2 \cdot z_{uu} - 2\, z_{uM} \cdot \frac{x^2}{y^2} + \frac{x^2}{y^4} \cdot z_{MM}}$

$A = x^2 \cdot z_{xx} - y^2 z_{yy} + x \cdot z_x - y \cdot z_y$

$A = \cancel{x^2 y^2 z_{uu}} + 2x^2 \cdot z_{uM} + \cancel{\frac{x^2}{y^2} \cdot z_{MM}} - \cancel{x^2 y^2 \cdot z_{uu}} + 2y^2 z_{uM} \cdot \frac{x^2}{y^2} - \cancel{\frac{x^2}{y^2} z_{MM}} +$

$+ \cancel{xy \cdot z_u} + \frac{x}{y} \cdot z_M - \cancel{xy \cdot z_u} + \frac{x}{y} z_M$

$\boxed{A = 4x^2 z_{uM} + \frac{2x}{y} z_M}$

(14.)

(15.)

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14. $z = z(x,y)$; $z = \varphi(x+y)\psi(x-y)$

$z \cdot z_{xx} - y^2 \cdot z_{yy} + x \cdot z_x - y \cdot z_y = ?$

$z = \varphi(u)\psi(M)$

$u = x+y \qquad M = x-y$

$u_x = 1 \qquad M_x = 1$

$u_y = 1 \qquad M_y = -1$

?

$z = \varphi(u)\psi(M)$

$z_x = \varphi_u u_x \cdot \psi(M) + \varphi(u) \cdot \psi_M M_x = \varphi_u \cdot \psi(M) + \varphi(u) \cdot \psi_M$

$z_{xx} = \varphi_{uu} \cdot u_x \cdot \psi(M) + \varphi_u \cdot \psi_M M_x$

15. $F\left(\frac{x}{y}, \frac{y}{z}\right) = 0$; $z = z(x,y)$; $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} - z = ?$

$x \cdot z_x + y \cdot z_y - z = ?$

$u = \frac{x}{y} \qquad M = \frac{y}{z}$

$u_x = \frac{1}{y} \qquad M_x = -\frac{y}{z^2} \cdot z_x \qquad F_x = 0$

$F_{xx} = 0$

$u_y = -\frac{x}{y^2} \qquad M_y = \frac{z - y \cdot z_y}{z^2}$

$F_x = 0$

$F_x = F_u \cdot u_x + F_M M_x = F_u \cdot \frac{1}{y} - F_M \cdot \frac{y}{z^2} \cdot z_x = 0$

$\frac{F_M \cdot y \cdot z_x}{z^2} = \frac{F_u}{y} \Rightarrow \boxed{z_x = \frac{F_u \cdot z^2}{F_M \cdot y^2}}$

$F_y = 0$

$F_y = F_u u_y + F_M M_y = -F_u \frac{x}{y^2} + F_M \cdot \frac{z - y \cdot z_y}{z^2}$

$\frac{F_M (z - y \cdot z_y)}{z^2} = F_u \cdot \frac{x}{y^2}$

$\frac{F_M \cdot z}{z^2} - \frac{F_M \cdot z_y}{z^2} = \frac{F_u \cdot x}{y^2}$

$\frac{F_M y \cdot z_y}{z^2} = \frac{F_M}{z} - \frac{F_u \cdot x}{y^2} = \frac{F_M \cdot y^2 - F_u x z}{y^2 \cdot z}$

$z_y = \frac{z^2 (F_M \cdot y^2 - F_u x z)}{y^3 \cdot z \cdot F_M} = \boxed{\frac{F_M y^2 \cdot z - F_u \cdot x \cdot z^2}{F_M y^3}}$

$A = x \cdot z_x + y \cdot z_y - z$

$A = \frac{F_u \cdot x \cdot z^2}{F_M \cdot y^2} + \frac{F_M y^2 z - F_u x z^2}{F_M y^2} - z$

$A = \frac{\cancel{F_u \cdot x \cdot z^2} - \cancel{F_u \cdot x \cdot z^2} + \cancel{F_M \cdot y \cdot z} - \cancel{zF_M y^3}}{F_M \cdot y^3} = 0$

(16) $F(x+y+z,\ x^2+2yz)=0\ ;\ z=z(x,y)$

$(y-z)\cdot z_x+(x-y)z_y-z+x=?$

$u=x+y+z \qquad M=x^2+2yz$

$u_x=1+z_x \qquad M_x=2x+2yz_x$

$u_y=1+z_y \qquad M_y=2z+2yz_y$

$F_x=F_u u_x+F_M M_x=F_u(1+z_x)+F_M\cdot 2(x+y\cdot z_x)=0$

$F_u+F_u\cdot z_x+2x\cdot F_M+2yF_M\cdot z_x=0$

$z_x(F_u+2yF_M)=-F_u-2xF_M$

$$\boxed{z_x=-\frac{F_u+2xF_M}{F_u+2yF_M}}$$

$F_y=F_u u_y+F_M M_y=F_u(1+z_y)+F_M(2z+2y\cdot z_y)=0$

$F_u+F_u\cdot z_y+2z\cdot F_M+2y\cdot F_M\cdot z_y=0$

$z_y(F_u+2yF_M)=-F_u-2zF_M$

$$\boxed{z_y=-\frac{F_u+2zF_M}{F_u+2yF_M}}$$

$A=(y-z)\cdot z_x+(x-y)z_y-z+x$

$$A=(z-y)\frac{F_u+2xF_M}{F_u+2y(F_M)}+(y-x)\cdot\frac{F_u+2zF_M}{F_u+2yF_M}-z+x=$$

$$=\frac{z(F_u+2xF_M)-y(F_u+2xF_M)+y(F_u+2zF_M)-x(F_u+2zF_M)-z(F_u+2yF_M)+x(F_u+2yF_M)}{F_u+2yF_M}$$

$$=\frac{z(\cancel{F_u}+2xF_M-\cancel{F_u}-2yF_M)+y(\cancel{F_u}+2zF_M-\cancel{F_u}-2xF_M)+x(\cancel{F_u}+2yF_M-\cancel{F_u}-2zF_M)}{F_u+2yF_M}=$$

$$=\frac{2zF_M(x-y)+2yF_M(z-x)+2xF_M(y-z)}{F_u+2yF_M}=$$

$$=\frac{2F_M\left(z(x-y)+y(z-x)+x(y-z)\right)}{F_u+2yF_M}=$$

$$=\frac{2F_M(\cancel{zx}-\cancel{zy}+\cancel{zy}-\cancel{yx}+\cancel{yx}-\cancel{zx})}{F_u+2yF_M}=0$$

$$\boxed{A=0}$$

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(10) $F\left(x^2+y^2, \frac{z}{x}\right)=0$, $z=z(x,y)$

$A = xy\cdot z_x - x^2\cdot z_y - y\cdot z = ?$

$u = x^2+y^2$ $\qquad$ $M = \frac{z}{x}$

$u_x = 2x$ $\qquad$ $M_x = \frac{z_x\cdot x - z}{x^2}$

$u_y = 2y$ $\qquad$ $M_y = \frac{z_y}{x}$

$F_x = F_u u_x + F_M M_x = F_u\cdot 2x + F_M\cdot\frac{x\cdot z_x - z}{x^2} = 0$

$F_u\cdot 2x + F_M\cdot\frac{x\cdot z_x - z}{x^2} = 0$

$\frac{F_M(x\cdot z_x - z)}{x^2} = -2xF_u$

$x\cdot z_x - z = -\frac{2x^3F_u}{F_M}$

$x z_x = z - \frac{2x^3F_u}{F_M}$

$$\boxed{z_x = \frac{z\cdot F_M - 2x^3F_u}{x\cdot F_M}}$$

$F_y = F_u u_y + F_M M_y = F_u\cdot 2y + F_M\cdot\frac{z_y}{x} = 0$

$\frac{F_M\cdot z_y}{x} = -F_u\cdot 2y$

$$\boxed{z_y = -\frac{2xyF_u}{F_M}}$$

$A = xy\cdot z_x - x^2 z_y - y\cdot z$

$A = \cancel{x}y\cdot\frac{z\cdot F_M - 2x^3F_u}{\cancel{x}F_M} + x^2\cdot\frac{2xyF_u}{F_M} - y\cdot z =$

$= \frac{\cancel{yzF_M} - \cancel{2yx^3F_u} + \cancel{2x^3yF_u} - \cancel{y\cdot zF_M}}{F_M} = 0$

$$\boxed{A=0}$$

(17) $z(x,y) = e^{y} \cdot f\left(y \cdot e^{\frac{x^2}{2y^2}}\right)$ $\qquad (x^2-y^2) z_x + xy z_y - xyz = ?$

$u = y \cdot e^{\frac{x^2}{2y^2}}$

$u_x = \cancel{y} \cdot e^{\frac{x^2}{2y^2}} \cdot \frac{x}{y^2} = \boxed{\frac{x}{y} \cdot e^{\frac{x^2}{2y^2}}}$

$u_y = e^{\frac{x^2}{2y^2}} + y \cdot e^{\frac{x^2}{2y^2}} \cdot \left(-\frac{2x^2}{2y^3}\right) = \boxed{e^{\frac{x^2}{2y^2}}\left(1 - \frac{x^2}{y^2}\right)}$

$z_x = e^{y} \cdot f_u \cdot u_x = e^{y} f_u \cdot \frac{x}{y} \cdot e^{\frac{x^2}{2y^2}} = \frac{x}{y} \cdot f_u \cdot e^{y+\frac{x^2}{2y^2}}$

$z_y = e^{y} \cdot f_u + e^{y} \cdot f_u \cdot u_y = e^{y} \cdot f_u \left(1 + e^{\frac{x^2}{2y^2}}\left(1 - \frac{x^2}{y^2}\right)\right) = f_u\left(e^{y} + e^{y+\frac{x^2}{2y^2}}\left(1 - \frac{x^2}{y^2}\right)\right)$

$z = e^{y} \cdot f_u$

ИСПРАВКА

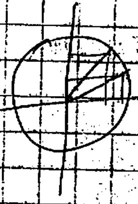
$A = (x^2-y^2) z_x + xy z_y - xyz$

$A = \frac{x(x^2-y^2)}{y} \cdot f_u \cdot e^{y+\frac{x^2}{2y^2}} + xy \cdot f_u \left(e^{y} + e^{y+\frac{x^2}{2y^2}}\left(1 - \frac{x^2}{y^2}\right)\right) - xy \cdot e^{y} f_u$

$A = \frac{x(x^2-y^2)}{y} \cdot f_u \cdot e^{y+\frac{x^2}{2y^2}} \cancel{+ e^{y} f_u \cdot xy} + xy \cdot f_u \cdot e^{y+\frac{x^2}{2y^2}} - \frac{x^3}{y} \cdot f_u \cdot e^{y+\frac{x^2}{2y^2}} \cancel{- xy e^{y} f_u}$

$A = \cancel{\frac{x^3}{y} f_u e^{y+\frac{x^2}{2y^2}}} \cancel{- xy \cdot f_u \cdot e^{y+\frac{x^2}{2y^2}}} \cancel{+ xy f_u e^{y+\frac{x^2}{2y^2}}} \cancel{- \frac{x^3}{y} f_u \cdot e^{y+\frac{x^2}{2y^2}}} = 0$

$\boxed{A = 0}$



$\cos 30° = \frac{\sqrt{3}}{2}$

$\sin 30° = \frac{1}{2}$

$\sin 60° = \frac{1}{2}$

$\cos 60° = \frac{\sqrt{3}}{2}$

(18) $\cos^2 x + \cos^2 y + \cos^2 z = \frac{9}{4}$; $z = z(x,y)$

$dz(M)$, $d^2 z(M)$, $M\left(\frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{6}\right)$

$\cos^2 x + \cos^2 y + \cos^2 z = \frac{9}{4} \Big/ \frac{\partial}{\partial x}$ $\qquad \cos^2 x + \cos^2 y + \cos^2 z = \frac{9}{4} \Big/ \frac{\partial}{\partial y}$

$-2\cos x \cdot \sin x - 2\cos z \cdot \sin z \cdot z_x = 0$ $\qquad -2\cos y \cdot \sin y - 2\cos z \cdot \sin z \cdot z_y = 0$

$2\cos z \cdot \sin z \cdot z_x = -2\cos x \sin x$ $\qquad 2\cos z \sin z \cdot z_y = -2\cos y \sin y$

$\boxed{z_x = -\frac{\sin 2x}{\sin 2z}} = -1$ $\qquad \boxed{z_y = -\frac{\sin 2y}{\sin 2z}} = -1$

$z_{xx} = \frac{(-\cos 2x \cdot 2)\sin 2z + \sin 2x(\cos 2z \cdot 2z_x)}{\sin^2 2z} =$

$= \frac{z_x \cdot 2\sin 2x \cdot \cos 2z - 2\sin 2z \cos 2x}{(\sin 2z)^2}$

$= \frac{-2(\sin 2x)^2 \cdot \cos 2z - 2(\sin 2z)^2 \cos 2x}{(\sin 2z)^3} = \frac{-\frac{1}{2}\cdot\frac{\sqrt{3}}{2} - \frac{1}{2}\cdot\frac{\sqrt{3}}{2}}{\frac{1}{8}} = \frac{-\frac{2\sqrt{3}}{4}}{\frac{1}{8}} = -\frac{16\sqrt{3}}{4} = \boxed{-4\sqrt{3}}$

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НЕДЕЉИВО

$$z_{xy} = \frac{\sin 2x}{(\sin 2z)^2} \cdot \cos 2z \cdot 2z_y =$$

$$= \frac{\sin 2x \cdot \cos 2z}{(\sin 2z)^2} \cdot 2\left(-\frac{\sin 2y}{\sin 2z}\right) = -\frac{2(\sin 2x)(\sin 2y)(\cos 2z)}{(\sin 2z)^3} = \frac{-\frac{1}{2}\frac{\sqrt{3}}{2}}{\frac{1}{8}} = -\frac{8\sqrt{3}}{4} = \boxed{-2\sqrt{3}}$$

$$z_{yy} = \frac{-\cos 2y \cdot 2(\sin 2z) + \sin 2y \cdot \cos 2z \cdot 2z_y}{(\sin 2z)^2} =$$

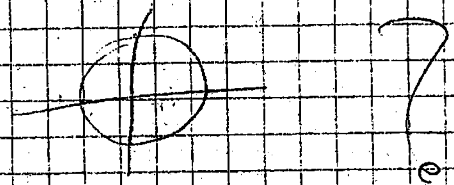
$$= \frac{-2\sin 2z \cdot \cos 2y - 2\sin 2y \cdot \cos 2z \cdot \frac{\sin 2y}{\sin 2z}}{(\sin 2z)^2} =$$

$$= \frac{-2(\sin 2z)^2 \cos 2y - 2(\sin 2y)^2 \cos 2z}{(\sin 2z)^3} = \frac{-\frac{1}{2}\frac{\sqrt{3}}{2} - \frac{1}{2}\cdot\frac{\sqrt{3}}{2}}{\frac{1}{8}} = \frac{-\frac{2\sqrt{3}}{4}}{\frac{1}{8}} = -\frac{16\sqrt{3}}{4} = \boxed{-4\sqrt{3}}$$

$$\boxed{dz(M) = -dx - dy}$$

$$\boxed{d^2z(M) = -4\sqrt{3}\,dx^2 - 4\sqrt{3}\,dx\,dy - 4\sqrt{3}\,dy^2}$$

(19) $\sin^2 x + \sin^2 y + \sin^2 z = 2; \quad z = z(x,y)$



$dz(M), d^2z(M), \quad M\left(\frac{\pi}{4}, \frac{\pi}{4}, \frac{\pi}{2}\right)$

НЕРЕШЕНО

$\sin^2 x + \sin^2 y + \sin^2 z = 2 \;/\; \frac{\partial}{\partial x}$

$2\sin x\cos x + 2\sin z\cos z \cdot z_x = 0$

$2\sin z \cdot \cos z \cdot z_x = -2\sin x\cos x$

$$\boxed{z_x = -\frac{\sin 2x}{\sin 2z}} = -\frac{1}{0} = -\infty$$

$\sin^2 x + \sin^2 y + \sin^2 z = 2 \;/\; \frac{\partial}{\partial y}$

$2\sin y \cdot \cos y + 2\sin z \cdot \cos z \cdot z_y = 0$

$2\sin z\cos z\, z_y = -2\sin y\cos y$

$$\boxed{z_y = -\frac{\sin 2y}{\sin 2z}} = -\infty$$

(20) $z(x,y) = g(x^2+y^2) \qquad z_{xx} + z_{yy} = ?$

$u = x^2 + y^2$

$$\boxed{\begin{aligned} u_x &= 2x \\ u_y &= 2y \end{aligned}}$$

$z_x = g_u \cdot u_x = g_u \cdot 2x$

$z_y = g_u \cdot u_y = g_u \cdot 2y$

$z_{xx} = g_{uu} \cdot u_x \cdot 2x + g_u \cdot 2 = g_{uu} \cdot 4x^2 + 2g_u$

$z_{yy} = g_{uu}\, u_y \cdot 2y + g_u \cdot 2 = g_{uu} \cdot 4y^2 + 2g_u$

$$\boxed{A = g_{uu}(4x^2 + 4y^2) + 4g_u}$$

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(21) $z(x,y) = g\left(\frac{x}{y}\right) \qquad z_{xx} + z_{yy} = ?$

$u = \frac{x}{y}$

$\boxed{u_x = \frac{1}{y} \qquad u_y = -\frac{x}{y^2}}$

$z_x = g_u u_x = g_u \cdot \frac{1}{y}$

$z_{xx} = g_{uu} \cdot u_x \cdot \frac{1}{y} = g_{uu} \cdot \frac{1}{y^2}$

$z_y = g_u u_y = -g_u \cdot \frac{x}{y^2}$

$z_{yy} = -g_{uu} \cdot u_y \cdot \frac{x}{y^2} - g_u \cdot \left(-\frac{2x}{y^3}\right) =$

$= -g_{uu} \cdot \left(-\frac{x}{y^2}\right) \cdot \frac{x}{y^2} + g_u \cdot \frac{2x}{y^3} =$

$= g_{uu} \cdot \frac{x^2}{y^4} + g_u \cdot \frac{2x}{y^3}$

$A = g_{uu} \cdot \frac{1}{y^2} + g_{uu} \frac{x^2}{y^4} + g_u \cdot \frac{2x}{y^3} =$

$= \boxed{g_{uu}\left(\frac{x^2+y^2}{y^4}\right) + g_u \cdot \frac{2x}{y^3}}$

(22) $f(x^2+y^2+z^2) = x+y+z \quad ; \; z = z(x,y)$

$A = (y+z)z_x + (z-x)z_y - x+y = ?$

$u = x^2+y^2+z^2$

$\boxed{\begin{aligned} u_x &= 2x + 2z \cdot z_x \\ u_y &= 2y + 2z \cdot z_y \end{aligned}}$

$f_u u_x = 1 + z_x$

$z_x = f_u u_x - 1 = f_u \cdot (2x + 2z \cdot z_x) - 1 = 2x \cdot f_u + 2z \cdot f_u \cdot z_x - 1$

$z_x - 2z \cdot f_u \cdot z_x = 2x f_u - 1$

$z_x (1 - 2z \cdot f_u) = 2x f_u - 1$

$\boxed{z_x = \frac{2x \cdot f_u - 1}{1 - 2z \cdot f_u}}$

$f_u \cdot u_y = 1 + z_y$

$z_y = f_u u_y - 1 = f_u(2y + 2z \cdot z_y) - 1 = 2y \cdot f_u + 2z \cdot f_u \cdot z_y - 1$

$$z_y - 2z \cdot f_u \cdot z_y = 2y f_u - 1$$

$$z_y(1 - 2z \cdot f_u) = 2y \cdot f_u - 1$$

$$\boxed{z_y = \frac{2y \cdot f_u - 1}{1 - 2z f_u}}$$

$$A = (y-z) z_x + (z-x) z_y - x + y$$

$$A = (y-z) \cdot \frac{2x f_u - 1}{1 - 2z \cdot f_u} + (z-x) \cdot \frac{2y f_u - 1}{1 - 2z f_u} - x + y =$$

$$= \frac{\cancel{2xy f_u} - y - 2xz f_u + z + 2zy f_u - \cancel{z} - \cancel{2xy f_u} + x}{1 - 2z f_u} - x + y =$$

$$= \frac{\cancel{-2xz \cdot f_u} + \cancel{2z \cdot y f_u} - \cancel{y} + \cancel{x} - \cancel{x} + \cancel{2xz f_u} + \cancel{y} - \cancel{2yz f_u}}{1 - 2z \cdot f_u} = 0$$

$$\boxed{A = 0}$$

(23.) $z = z(x,y)$; $x = s\cos e$; $y = s\sin e$

$$A = x z_x + y z_y = ?$$

$$z = z(x,y) = z(s\cos e, s\sin e) \qquad z = z(u,v) \quad \begin{matrix} u = u(x,y) \\ v = v(x,y) \end{matrix}$$

$$x = s\cos e \qquad y = s\sin e \qquad z_x = z_u u_x + z_v v_x$$

$$x_s = \cos e \qquad y_s = \sin e \qquad z_s = z_x x_s + z_y y_s$$

$$x_e = -s\sin e \qquad y_e = s\cos e$$

$$z_s = z_x x_s + z_y y_s = z_x \cdot \cos e + z_y \cdot \sin e$$

$$z_e = z_x x_e + z_y y_e = -z_x s \sin e + z_y \cdot s\cos e$$

$$z_x \cos e + z_y \sin e = z_s \quad / (+s\sin e)$$

$$-z_x \cdot s\sin e + z_y s\cos e = z_e \quad / \cos e$$

$$+z_x s\sin e \cos e + z_y s(\sin e)^2 = +z_s \cdot s \cdot \sin e$$

$$-z_x \cdot s\sin e \cos e + z_y s(\cos e)^2 = z_e \cdot \cos e$$

$$z_y \cdot s(\sin^2 e + \cos^2 e) = z_s \cdot s \cdot \sin e + z_e \cdot \cos e$$

$$\boxed{z_y = \frac{z_s \cdot s \cdot \sin e + z_e \cdot \cos e}{s}}$$

$$z_x \cos e = z_s - z_y \sin e$$

$$z_x = \frac{z_s - z_y \cdot \sin e}{\cos e}$$

$$Z_x = \frac{Z_s - \frac{Z_s \cdot s \cdot \sin e + Z_e \cdot \cos e}{s} \cdot \sin e}{\cos e}$$

$$= \boxed{\frac{Z_s \cdot s - Z_s \cdot s \cdot \sin^2 e - Z_e \cdot \sin e \cos e}{s \cos e}}$$

$$A = \cancel{s\cos e} \cdot \frac{Z_s \cdot s - Z_s \cdot s \sin^2 e + Z_e \sin e \cos e}{\cancel{s \cos e}} + s \sin e \cdot \frac{Z_s \cdot s \cdot \sin e + Z_e \cos e}{s}$$

$$= Z_s \cdot s - \cancel{Z_s \cdot s \sin^2 e} - \cancel{Z_e \sin e \cos e} + \cancel{Z_s \cdot s \sin^2 e} + \cancel{Z_e \cdot \sin e \cos e}$$

$$\boxed{A = s \cdot Z_s}$$

(24)

$$\boxed{\begin{aligned} Z_x &= \frac{Z_s \cdot s - Z_s \cdot s \cdot \sin^2 e - Z_e \cdot \sin e \cdot \cos e}{s \cos e} \\ Z_y &= \frac{Z_s \cdot s \cdot \sin e + Z_e \cos e}{s} \end{aligned}}$$

$$\boxed{\begin{aligned} x &= s \cos e \\ y &= s \sin e \end{aligned}}$$

$x Z_y - y Z_x = ?$

$$A = \cancel{s}\cos e \cdot \frac{Z_s \cdot s \cdot \sin e + Z_e \cdot \cos e}{\cancel{s}} - \cancel{s}\sin e \cdot \frac{Z_s \cdot s - Z_s \cdot s \cdot \sin^2 e - Z_e \sin e \cos e}{\cancel{s}\cos e}$$

$$A = \frac{Z_s \cdot s \cdot \sin e \cos^2 e + Z_e \cos^3 e - Z_s \cdot s \cdot \sin e + Z_s \cdot s \sin^3 e - Z_e \cdot \sin^2 e \cos e}{\cos e}$$

$$A = \frac{s \cdot \sin e \cdot \cos e (Z_s \cancel{\cos e} - Z_e \cdot \sin e) + Z_e \cos^3 e - Z_s \cdot s \cdot \sin e \overbrace{(1 - \sin^2 e)}^{\cos^2 e}}{\cos e}$$

$$A = \frac{-Z_e \cdot s \cdot \sin^2 e \cdot \cos e + Z_e \cos^3 e}{\cos e} = \frac{Z_e \cancel{\cos e}(\cos^2 e - s \sin^2 e)}{\cancel{\cos e}} \Rightarrow$$

$$\boxed{A = Z_e (\cos^2 e - s \sin^2 e)}$$

(25) $Z_s = Z_x \cdot \cos e + Z_y \sin e$

$$Z_{ss} = (Z_{xx} X_s + Z_{xy} Y_s) \cos e + (Z_{yy} Y_s + Z_{yx} X_s) \sin e =$$

$$= (Z_{xx} \cdot \cos e + Z_{xy} \cdot \sin e) \cos e + (Z_{yy} \cdot \sin e + Z_{xy} \cdot \cos e) \sin e =$$

$$= \boxed{Z_{xx} \cdot \cos^2 e + 2 Z_{xy} \cdot \sin e \cos e + Z_{yy} \sin^2 e}$$

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$$\boxed{Z_{se} \cdot s \cdot \cos e = -Z_{xx} \cdot s^2 \sin e \cos^2 e + Z_{xy} \cdot s^2 \cos e (\cos^2 e - \sin^2 e) + Z_{yy} \cdot s^2 \sin e \cos^2 e - Z_s \cdot \cos e}$$

$$z_\theta = -z_x\rho\sin\theta + z_y\rho\cos\theta$$

$$z_{\theta\theta} = -\rho((z_{xx}x_\theta + z_{xy}y_\theta)\sin\theta + z_x\cos\theta) + \rho((z_{yy}y_\theta + z_{yx}x_\theta)\cos\theta - z_y\cdot\sin\theta) =$$

$$= \rho((z_{yy}\rho\cos\theta - z_{yx}\rho\sin\theta)\cos\theta - z_y\sin\theta - (z_{xx}\cdot(-\rho\sin\theta) + z_{xy}\cdot\rho\cos\theta)\sin\theta + z_x\cos\theta) =$$

$$= \rho(z_{yy}\cdot\rho\cdot\cos^2\theta - z_{yx}\cdot\rho\sin\theta\cos\theta - z_y\sin\theta + z_{xx}\rho\sin^2\theta - z_{xy}\rho\sin\theta\cos\theta - z_x\cdot\cos\theta) =$$

$$z_{\theta\theta} = \boxed{\rho(z_{yy}\cdot\rho\cos^2\theta - 2z_{xy}\cdot\rho\sin\theta\cos\theta + z_{xx}\cdot\rho\sin^2\theta - z_y\cdot\sin\theta - z_x\cdot\cos\theta)}$$

$$z_{\rho\theta} = (z_{xx}x_\theta + z_{xy}y_\theta)\cos\theta - z_x\cdot\sin\theta + (z_{yy}y_\theta + z_{xy}x_\theta)\sin\theta + z_y\cdot\cos\theta =$$

$$= (z_{xx}\cdot(-\rho\sin\theta) + z_{xy}\cdot\rho\cos\theta)\cos\theta - z_x\sin\theta + (z_{yy}\cdot\rho\cos\theta + z_{xy}\cdot(-\rho\sin\theta))\sin\theta + z_y\cdot\cos\theta =$$

$$= -z_{xx}\cdot\rho\sin\theta\cos\theta + z_{xy}\cdot\rho\cos^2\theta - z_x\sin\theta + z_{yy}\cdot\rho\cdot\sin\theta\cos\theta - z_{xy}\cdot\rho\sin^2\theta + z_y\cdot\cos\theta =$$

$$= \boxed{-z_{xx}\rho\sin\theta\cos\theta + z_{xy}\rho(\cos^2\theta - \sin^2\theta) + z_{yy}\rho\sin\theta\cos\theta - z_x\cdot\sin\theta + z_y\cdot\cos\theta}$$

$$z_{\rho\rho} = z_{xx}\cdot\cos^2\theta + 2z_{xy}\sin\theta\cos\theta + z_{yy}\sin^2\theta$$

$$z_{\theta\theta} = z_{xx}\cdot\rho^2\sin^2\theta - 2z_{xy}\rho^2\sin\theta\cos\theta + z_{yy}\cdot\rho^2\cos^2\theta - z_x\cdot\rho\cos\theta - z_y\cdot\rho\sin\theta$$

$$z_{\rho\theta} = -z_{xx}\cdot\rho\sin\theta\cos\theta + z_{xy}\rho(\cos^2\theta - \sin^2\theta) + z_{yy}\rho\sin\theta\cos\theta - z_x\sin\theta + z_y\cdot\cos\theta$$

$$z_{\theta\theta} = z_{xx}\rho^2\sin^2\theta - 2z_{xy}\rho^2\sin\theta\cos\theta + z_{yy}\rho^2\cos^2\theta - \cancel{\rho\cos\theta}\cdot\frac{z_\rho\rho - z_\rho\rho\sin^2\theta - z_\theta\sin\theta\cos\theta}{\cancel{\rho\cos\theta}} -$$

$$- \cancel{\rho}\sin\theta\cdot\frac{z_\rho\rho\sin\theta + z_\theta\cos\theta}{\cancel{\rho}}$$

$$z_{\theta\theta} = z_{xx}\rho^2\sin^2\theta - 2z_{xy}\rho^2\sin\theta\cos\theta + z_{yy}\rho^2\cos^2\theta - z_\rho\rho - z_\rho\rho\sin^2\theta - z_\theta\sin\theta\cos\theta -$$

$$- z_\rho\rho\sin^2\theta - z_\theta\sin\theta\cos\theta$$

$$\boxed{z_{\theta\theta} = z_{xx}\rho^2\sin^2\theta - 2z_{xy}\rho^2\sin\theta\cos\theta + z_{yy}\rho^2\cos^2\theta - z_\rho\rho - 2z_\rho\rho\sin^2\theta - 2z_\theta\sin\theta\cos\theta}$$

$$z_{\rho\theta} = -z_{xx}\cdot\rho\sin\theta\cos\theta + z_{xy}\rho(\cos^2\theta - \sin^2\theta) + z_{yy}\cdot\rho\sin\theta\cos\theta -$$

$$- \sin\theta\cdot\frac{z_\rho\rho - z_\rho\rho\sin^2\theta - z_\theta\sin\theta\cos\theta}{\rho\cos\theta} + \cos\theta\cdot\frac{z_\rho\rho\sin\theta + z_\theta\cos\theta}{\rho}$$

$$z_{\rho\theta} = -z_{xx}\rho^2\sin\theta\cos^2\theta + z_{xy}\rho^2\cos\theta(\cos^2\theta - \sin^2\theta) + z_{yy}\rho^2\sin\theta\cos^2\theta -$$

$$\frac{- z_\rho\rho\sin\theta + z_\rho\rho\sin^3\theta + z_\theta\sin^2\theta\cos\theta + z_\rho\rho\sin\theta\cos^2\theta + z_\theta\cos^3\theta}{\rho\cos\theta}$$

$$z_{\rho\theta} = -z_{xx}\rho^2\sin\theta\cos^2\theta + z_{xy}\rho^2\cos\theta(\cos^2\theta - \sin^2\theta) + z_{yy}\rho^2\sin\theta\cos^2\theta -$$

$$\frac{-\cancel{z_\rho\rho\sin\theta} + \cancel{z_\rho\rho\sin\theta}(\sin^2\theta + \cos^2\theta) + z_\theta\cos\theta(\sin^2\theta + \cos^2\theta)}{\rho\cos\theta}$$

1° $z_{\rho\rho} = z_{xx}\cdot\cos^2\varphi + 2\cdot z_{xy}\cdot\sin\varphi\cos\varphi + z_{yy}\cdot\sin^2\varphi$

2° $z_{\varphi\varphi} = z_{xx}\,\rho^2\cdot\sin^2\varphi - 2 z_{xy}\cdot\rho^2\sin\varphi\cos\varphi + z_{yy}\cdot\rho^2\cos^2\varphi - z_\rho\cdot\rho - 2 z_\rho\,\rho\,\sin^2\varphi - 2\,z_\varphi\cdot\sin\varphi\cos\varphi$

3° $z_{\rho\varphi}\cdot\rho\cos\varphi = -z_{xx}\,\rho^2\sin\varphi\cos^2\varphi + z_{xy}\cdot\rho^2\cos\varphi(\cos^2\varphi - \sin^2\varphi) + z_{yy}\cdot\rho^2\sin\varphi\cos^2\varphi - z_\rho\cdot\cos\varphi$

25, 26 $\to$?

$\cancel{26}$ $z = z(x,y)$

$$\frac{x-1}{z+1} = \varphi\left(\frac{y-1}{z+1}\right) \qquad z_{xx}\cdot z_{yy} - z^2_{xy} = ?$$

$$u = \frac{y-1}{z+1}$$

$$u_x = -\frac{y-1}{(z+1)^2}\cdot z_x = \frac{z_x(1-y)}{(z+1)^2} \Rightarrow \frac{-(z+1)}{1-x+\varphi'\cdot y-\varphi'}\cdot\frac{(1-y)}{(z+1)^2} = \boxed{\frac{y-1}{1-x+\varphi'\cdot y-\varphi'}}$$

$$u_y = \frac{z+1-(y-1)\cdot z_y}{(z+1)^2} = \frac{z_y(1-y)+z+1}{(z+1)^2} \Rightarrow \frac{\frac{\varphi'(z+1)}{1-x-\varphi'(1-y)}(1-y)+z+1}{(z+1)^2}$$

$$\frac{x-1}{z+1} = \varphi(u) \;\Big/ \frac{\partial}{\partial x}$$

$$u_y = \frac{\varphi'(z+1)(1-y)+(z+1)(1-x-\varphi'(1-y))}{(z+1)^2(1-x-\varphi'(1-y))}$$

$$\frac{z+1-(x-1)\cdot z_x}{(z+1)^2} = \varphi' u_x = \varphi'\cdot\frac{z_x(1-y)}{(z+1)^2}$$

$$= \frac{\cancel{(z+1)}(\cancel{\varphi'(1-y)}+1-x-\cancel{\varphi'(1-y)})}{(z+1)^{\cancel{2}}(1-x-\varphi'(1-y))}$$

$$z_x(1-x)+z+1+\varphi' z_x(y-1) = 0$$

$$z_x(1-x+\varphi'\cdot y-\varphi') = -z-1$$

$$\boxed{z_x = \frac{-1-z}{1-x+\varphi'\cdot y-\varphi'}} \qquad \boxed{u_y = \frac{1-x}{(z+1)(1-x-\varphi'(1-y))}}$$

$$z_{xx} = \frac{-z_x(1-x+\varphi' y-\varphi')+z(-1+y\,\varphi'' u_x-\varphi'' u_x)}{(1-x+\varphi'\cdot y-\varphi')^2} =$$

$$= \frac{\frac{1+z}{\cancel{1-x+\varphi' y-\varphi'}}\cancel{(1-x+\varphi' y-\varphi')}+z\left(-1+(y\,\varphi''-1)\cdot\frac{y-1}{1-x+\varphi'\cdot y-\varphi'}\right)}{(1-x+\varphi'\cdot y-\varphi')^2} =$$

$$= \frac{1+\cancel{z}-\cancel{z}+\frac{z(y-1)(y\,\varphi''-1)}{1-x+\varphi' y-\varphi'}}{(1-x+\varphi' y-\varphi')^2} = \frac{1-x+\varphi'\cdot y-\varphi'+z(y-1)(y\,\varphi''-1)}{(1-x+\varphi'\cdot y-\varphi')^3}$$

$$-\frac{x-1}{(z+1)^2}\cdot z_y = \varphi'\cdot u_y \Rightarrow$$

$$\frac{1-x}{(z+1)^2}\,z_y = \frac{\varphi'(z_y(1-y)+z+1)}{(z+1)^2} =$$

$$z_y(1-x) = z_y \cdot c_u(1-y) + c_u(z+1)$$

$$z_y(1-x) - z_y \cdot c_u(1-y) = c_u(z+1)$$

$$z_y(1-x-c_u(1-y)) = c_u(z+1)$$

$$\boxed{z_y = \frac{c_u(z+1)}{1-x-c_u(1-y)}}$$

$$z_{yy} = \frac{[(c_{uu}\cdot u_y)\cdot(z+1) + c_u\cdot z_y](1-x-c_u(1-y)) - c_u(z+1)[-c_{uu}\cdot u_y(1-y) - c_u(-1)]}{(1-x-c_u(1-y))^2} =$$

$$= \frac{\left(c_{uu}\cdot\frac{c_u(\cancel{z+1})(1-y)+(\cancel{z+1})(1-x-\cancel{c_u(1-y)})}{(\cancel{z+1})^2(1-x-c_u(1-y))}\cdot(\cancel{z+1}) + c_u\cdot\frac{1-x}{(z+1)(1-x-c_u(1-y))}\right)\cdot(1-x-c_u(1-y)) - c_u(\cancel{z+1})\left(-c_{uu}(1-y)\cdot\frac{1-x}{(\cancel{z+1})(1-x-c_u(1-y))}+\frac{c_u}{z+1}\right)}{(1-x-c_u(1-y))^2} =$$

$$z_{yy} = \frac{\left(\frac{c_{uu}(1-x)}{(1-x-c_u(1-y))} + \frac{c_u(1-x)}{(z+1)(1-x-c_u(1-y))}\right)\cdot(1-x-c_u(1-y)) + \frac{c_u\cdot c_{uu}(1-y)(1-x)}{(1-x-c_u(1-y))} - \frac{c_u^2}{z+1}}{\cancel{(1-x-c_u(1-y))^2}}$$

$$z_{yy} = c_{uu}(1-x) + \frac{c_u(1-x)}{z+1} + \frac{c_u\cdot c_{uu}(1-y)(1-x)}{1-x-c_u(1-y)} - \frac{c_u^2}{z+1} =$$

$$= \frac{c_{uu}(1-x)(1-x-\cancel{c_u(1-y)}+\cancel{c_u(1-y)})}{1-x-c_u(1-y)} + \frac{c_u(1-x-c_u)}{z+1}$$

$$= \frac{c_{uu}(1-x)^2}{1-x-c_u(1-y)} + \frac{c_u(1-x-c_u)}{z+1} = \boxed{\frac{c_{uu}(1-x)^2(z+1) + c_u(1-x-c_u)(1-x-c_u(1-y))}{(z+1)(1-x-c_u(1-y))}}$$

$$z_{xy} = \frac{z_y(1-x-c_u(1-y)) + z(-c_{uu}\cdot u_y(1-y) - c_u(-1))}{(1-x-c_u(1-y))^2} =$$

$$= \frac{\frac{c_u(z+1)(\cancel{1-x-c_u(1-y)})}{\cancel{1-x-c_u(1-y)}} - z\left(c_{uu}(1-y)\cdot\frac{1-x}{(z+1)(1-x-c_u(1-y))} - c_u\right)}{(1-x-c_u(1-y))^2} =$$

$$= \frac{c_u(z+1) - \frac{c_{uu}(1-x)(1-y)\cdot z}{(z+1)(1-x-c_u(1-y))} - c_u\cdot z}{(1-x-c_u(1-y))^2} =$$

$$= \frac{c_u(z+1)^2(1-x-c_u(1-y)) - c_{uu}(1-x)(1-y)\cdot z - c_u\, z(z+1)(1-x-c_u(1-y))}{(z+1)(1-x-c_u(1-y))^3}$$

$$= \frac{c_u(z+1)(1-x-c_u(1-y))(z+1-z) - c_{uu}(1-x)(1-y)\cdot z}{(z+1)(1-x-c_u(1-y))^3} = \boxed{\frac{c_u(z+1)(1-x-c_u(1-y)) - c_{uu}(1-x)(1-y)\,z}{(z+1)(1-x-c_u(1-y))^3}}$$

(27.) $z = z(x,y)$; $\qquad z_{xx}\cdot z_{yy} - z_{xy}^2 = ?$

$$\frac{x-1}{z+1} = \varphi\left(\frac{y-1}{z+1}\right)$$

$$u = \frac{y-1}{z+1} \qquad u_x = -\frac{y-1}{(z+1)^2}\cdot z_x = \frac{-z_x(y-1)}{(z+1)^2}$$

$$u_y = \frac{z+1-(y-1)z_y}{(z+1)^2}$$

$$\frac{x-1}{z+1} = \varphi(u) \Big/ \frac{\partial}{\partial x}$$

$$\frac{1(z+1)-(x-1)\cdot z_x}{(z+1)^2} = \varphi_u\cdot u_x$$

$$\frac{z+1-(x-1)\cdot z_x}{(z+1)^2} = \varphi_u\cdot\frac{-z_x(y-1)}{(z+1)^2}$$

$$z+1-(x-1)z_x = -z_x\cdot\varphi_u(y-1)$$

$$z_x(\varphi_u(y-1)) - z_x(x-1) = -z-1$$

$$z_x(\varphi_u(y-1)-x+1) = -z-1$$

$$\boxed{z_x = \frac{-1-z}{1-x+\varphi_u(y-1)}}$$

$$u_x = \frac{-1-z}{1-x+\varphi_u(y-1)}\cdot\frac{(y-1)}{(z+1)^2} = \frac{y-1}{(z+1)(1-x+\varphi_u(y-1))}$$

$$\boxed{u_x = \frac{y-1}{(z+1)(1-x+\varphi_u(y-1))}}$$

$$z_{xx} = \frac{-z_x(1-x+\varphi_u(y-1))+(z+1)(-1+\varphi_{uu}u_x(y-1))}{(1-x+\varphi_u(y-1))^2} =$$

$$= \frac{\frac{z+1}{\cancel{1-x+\varphi_u(y-1)}}\cancel{(1-x+\varphi_u(y-1))} + (z+1)\left(\varphi_{uu}(y-1)\cdot\frac{y-1}{(z+1)(1-x+\varphi_u(y-1))} - 1\right)}{(1-x+\varphi_u(y-1))^2}$$

$$= \frac{\cancel{z+1} + \frac{\varphi_{uu}(y-1)}{(1-x+\varphi_u(y-1))} \cancel{-z-1}}{(1-x+\varphi_u(y-1))^2} = \frac{\varphi_{uu}(y-1)}{(1-x+\varphi_u(y-1))^3}$$

$$\boxed{z_{xx} = \frac{\varphi_{uu}(y-1)}{(1-x+\varphi_u(y-1))^3}}$$

$$\frac{x-1}{z+1} = \varphi(u) \Big/ \frac{\partial}{\partial y}$$

$$-\frac{x-1}{(z+1)^2}z_y = \varphi_u\cdot u_y$$

$$\frac{-z_y(x-1)}{(z+1)^2} = \varphi_u\cdot\frac{z+1-(y-1)z_y}{(z+1)^2}$$

$$-z_y(x-1) = \varphi_u(z+1) - \varphi_u(y-1)z_y$$

$$z_y(\varphi_u(y-1)) - z_y(x-1) = \varphi_u(z+1)$$

$$z_y(1-x+\varphi_u(y-1)) = \varphi_u(z+1)$$

$$\boxed{z_y = \frac{\varphi_u(z+1)}{1-x+\varphi_u(y-1)}}$$

$$u_y = \frac{z+1-(y-1)\frac{\varphi_u(z+1)}{1-x+\varphi_u(y-1)}}{(z+1)^2}$$

$$= \frac{(z+1)(1-x+\varphi_u(y-1)) - \varphi_u(y-1)(z+1)}{(z+1)^2(1-x+\varphi_u(y-1))}$$

$$= \frac{\cancel{(z+1)}(1-x+\cancel{\varphi_u(y-1)}-\cancel{\varphi_u(y-1)})}{(z+1)^{\cancel{2}}(1-x+\varphi_u(y-1))} = \frac{1-x}{(z+1)(1-x+\varphi_u(y-1))}$$

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$$\boxed{u_y = \frac{1-x}{(z+1)(1-x+C_u(y-1))}}$$

$$z_{yy} = \frac{(C_{uu} u_y (z+1) + C_u z_y)(1-x+C_u(y-1)) - C_u(z+1)(C_{uu}u_y(y-1)+C_u)}{(1-x+C_u(y-1))^2} =$$

$$= \frac{\left(C_{uu}\frac{1-x}{(z+1)(1-x+C_u(y-1))}(z+1) + C_u\frac{C_u(z+1)}{1-x+C_u(y-1)}\right)(1-x+C_u(y-1)) - C_u(z+1)\left(C_{uu}(y-1)\cdot\frac{1-x}{(z+1)(1-x+C_u(y-1))}+C_u\right)}{(1-x+C_u(y-1))^2} =$$

$$= \frac{\frac{C_{uu}(1-x)(z+1)+C_u^2(z+1)^2}{(z+1)\cancel{(1-x+C_u(y-1))}}\cdot\cancel{(1-x+C_u(y-1))} - C_u(z+1)\left(\frac{C_{uu}(y-1)(1-x)}{(z+1)(1-x+C_u(y-1))}+C_u\right)}{(1-x+C_u(y-1))^2} =$$

$$= \frac{\frac{\cancel{(z+1)}(C_{uu}(1-x)+C_u^2(z+1))}{\cancel{z+1}} - \frac{C_uC_{uu}(y-1)(1-x)}{(1-x+C_u(y-1))} - C_u^2(z+1)}{(1-x+C_u(y-1))^2} =$$

$$= \frac{[C_{uu}(1-x)+C_u^2(z+1)](1-x+C_u(y-1)) - C_uC_{uu}(y-1)(1-x) - C_u^2(z+1)(1-x+C_u(y-1))}{(1-x+C_u(y-1))^3}$$

$$= \frac{(1-x+C_u(y-1))(C_{uu}(1-x)+\cancel{C_u^2(z+1)}-\cancel{C_u^2(z+1)}) - C_uC_{uu}(y-1)(1-x)}{(1-x+C_u(y-1))^3} =$$

$$= \frac{C_{uu}(1-x)(1-x+\cancel{C_u(y-1)}-\cancel{C_u(y-1)})}{(1-x+C_u(y-1))^3} = \frac{C_{uu}(1-x)^2}{(1-x+C_u(y-1))^3}$$

$$\boxed{z_{yy} = \frac{C_{uu}(1-x)^2}{(1-x+C_u(y-1))^3}}$$

$$z_{yx} = \frac{(C_{uu}u_x(z+1)+C_u z_x)(1-x+C_u(y-1)) - C_u(z+1)(-1+C_{uu}u_x(y-1))}{(1-x+C_u(y-1))^2} =$$

$$= \frac{\left(C_{uu}\frac{y-1}{(z+1)(1-x+C_u(y-1))}(z+1) + C_u\cdot\frac{-1-z}{1-x+C_u(y-1)}\right)\cdot(1-x+C_u(y-1)) - C_u(z+1)\left(C_{uu}(y-1)\cdot\frac{y-1}{(z+1)(1-x+C_u(y-1))} - 1\right)}{(1-x+C_u(y-1))^2}$$

$$= \frac{\frac{C_{uu}(y-1)(z+1) - C_u(z+1)^2}{(z+1)\cancel{(1-x+C_u(y-1))}}\cdot\cancel{(1-x+C_u(y-1))} - \frac{C_uC_{uu}\cancel{(z+1)}(y-1)^2}{\cancel{(z+1)}(1-x+C_u(y-1))} + C_u(z+1)}{(1-x+C_u(y-1))^2}$$

$$= \frac{\frac{\cancel{(z+1)}(C_{uu}(y-1)-C_u(z+1))}{\cancel{z+1}} - \frac{C_uC_{uu}(y-1)^2}{(1-x+C_u(y-1))} + C_u(z+1)}{(1-x+C_u(y-1))^2}$$

$$= \frac{(C_{uu}(y-1)-C_u(z+1))(1-x+C_u(y-1)) - C_uC_{uu}(y-1)^2 + C_u(z+1)(1-x+C_u(y-1))}{(1-x+C_u(y-1))^3}$$

$$Z_{yx} = \frac{(1-x+\ell u(y-1))(\ell uu(y-1) - \cancel{\ell u(z+1)} + \cancel{\ell u(z+1)}) - \ell u\, \ell uu(y-1)^2}{(1-x+\ell u(y-1))^3}$$

$$= \frac{(1-x+\ell u(y-1))\cdot \ell uu(y-1) - \ell u\,\ell uu(y-1)^2}{(1-x+\ell u(y-1))^3}$$

$$= \frac{\ell uu(y-1)(1-x+\cancel{\ell u(y-1)}-\cancel{\ell u(y-1)})}{(1-x+\ell u(y-1))^3} = \frac{\ell uu(y-1)(1-x)}{(1-x+\ell u(y-1))^3}$$

$$\boxed{Z_{xy} = \frac{\ell uu(y-1)(1-x)}{(1-x+\ell u(y-1))^3}}$$

$$A = Z_{xx}\cdot Z_{yy} - Z_{xy}^2$$

$$A = \frac{\ell uu(y-1)}{(1-x+\ell u(y-1))^3}\cdot\frac{\ell uu(1-x)^2}{(1-x+\ell u(y-1))^3} - \frac{\ell uu^2(y-1)(1-x)}{(1-x+\ell u(y-1))^6}$$

$$= \frac{\ell uu^2(y-1)(1-x)}{(1-x+\ell u(y-1))^6}(\cancel{1}-x-\cancel{1}) = \boxed{-\frac{x\cdot\ell uu^2(y-1)(1-x)}{(1-x+\ell u(y-1))^6}}$$

(28.) $z = z(u,v)$; $u = x+y$; $v = xy$ $\quad Z_{xy} = ?$

$u = x+y \qquad M = xy$

$u_x = 1 \qquad M_x = y$

$u_y = 1 \qquad M_y = x$

$$Z_x = Z_u u_x + Z_M M_x = Z_u + y\cdot Z_M$$

$$Z_y = Z_u u_y + Z_M M_y = Z_u + x\cdot Z_M$$

$$Z_{xy} = Z_{uu}u_y + Z_{uM}M_y + Z_M + y\cdot(Z_{MM}M_y + Z_{Mu}u_y) =$$

$$= Z_{uu} + Z_{uM}\cdot x + Z_M + y(Z_{MM}\cdot x + Z_{Mu}) =$$

$$= Z_{uu} + x\cdot Z_{uM} + Z_M + xy\cdot Z_{MM} + yZ_{Mu} =$$

$$= Z_{uu} + (x+y)Z_{uM} + xyZ_{MM} + Z_M$$

(29.) $z = z(u,v)$; $u = x^2+y^2$; $v = x-y$ $\quad Z_x - Z_y = ?$

$u = x^2+y^2 \qquad M = x-y$

$u_x = 2x \qquad M_x = 1$

$u_y = 2y \qquad M_y = -1$

(30)

(31)

$$z_x = z_u u_x + z_v v_x = 2x\cdot z_u + z_v$$

$$z_y = z_u u_y + z_v v_y = 2y\cdot z_u - z_v$$

$$z_x - z_y = 2x z_u + z_v - 2y z_u + z_v =$$

$$\boxed{= 2(x-y)\cdot z_u + 2 z_v}$$

30) $z = z(u,v)$; $u = x$, $v = x^2+y^2$ $\qquad A = y z_x - x z_y = ?$

$u = x \qquad v = x^2+y^2$

$u_x = 1 \qquad v_x = 2x$

$u_y = 0 \qquad v_y = 2y$

$$z_x = z_u u_x + z_v v_x = z_u + z_v\cdot 2x$$

$$z_y = z_u u_y + z_v v_y = z_v\cdot 2y$$

$$A = y\cdot z_x - x z_y = y(z_u + z_v\cdot 2x) - x\cdot z_v\cdot 2y =$$

$$= y\cdot z_u + \cancel{2xy z_v} - \cancel{2xy z_v} =$$

$$\boxed{= y\cdot z_u}$$

31) $z = z(u,v)$; $u = \ln x$, $v = \ln\left(y+\sqrt{1+y^2}\right)$ $\qquad x z_x + \sqrt{1+y^2}\cdot z_y - xy = ?$

$u = \ln x \qquad v = \ln\left(y+\sqrt{1+y^2}\right)$

$\boxed{u_x = \frac{1}{x}} \qquad v_x = 0$

$u_y = 0 \qquad v_y = \frac{1}{y+\sqrt{1+y^2}}\left(1+\frac{1}{2\sqrt{1+y^2}}\cdot 2y\right) = \frac{1+\frac{y}{\sqrt{1+y^2}}}{y+\sqrt{1+y^2}} = \boxed{\frac{y+\sqrt{1+y^2}}{\sqrt{1+y^2}\,\left(y+\sqrt{1+y^2}\right)}}$

$$v_y = \frac{y+\sqrt{1+y^2}}{y\sqrt{1+y^2}+1+y^2} = \cancel{\frac{y+\sqrt{1+y^2}}{1+y^2+y\sqrt{1+y^2}}}$$ ~~НЕ ТРЕБА~~

$$z_x = z_u u_x + z_v \cancel{v_x}^{0} = z_u\cdot\frac{1}{x}$$

$$z_y = z_u \cancel{u_y}^{0} + z_v v_y = z_v\cdot\frac{y+\sqrt{1+y^2}}{1+y^2+y\sqrt{1+y^2}}$$

$$A = x\cdot z_x + z_y\sqrt{1+y^2} - xy$$

$$A = x\cdot\frac{1}{x}\cdot z_u + \cancel{\sqrt{1+y^2}}\cdot\frac{y+\sqrt{1+y^2}}{\cancel{\sqrt{1+y^2}}\left(y+\sqrt{1+y^2}\right)}z_v - xy = \boxed{z_u + z_v - xy}$$

(32) $z=z(u,v)$, $u=2x-z^2$, $v=\frac{y}{z}$ $\quad xz\cdot z_x+yz\cdot z_y=1$

$u=2x-z^2 \qquad v=\frac{y}{z}$

$u_x=2-2z\cdot z_x \qquad v_x=-\frac{y}{z^2}\cdot z_x$

$u_y=-2z\cdot z_y \qquad v_y=\frac{z-y\cdot z_y}{z^2}$

$z_x=z_u u_x+z_v v_x = z_u(2-2z\cdot z_x)+z_v\left(-\frac{y\cdot z_x}{z^2}\right)$

$z_x= 2z_u-2\cdot z\cdot z_u\cdot z_x-\frac{y\cdot z_v\cdot z_x}{z^2}$

$z\cdot z_x= 2\cdot z^2\cdot z_u-2\cdot z^3\cdot z_u\cdot z_x-y\,z_v\,z_x$

$z_x\cdot z^2+2z^3 z_u\cdot z_x+z_x\cdot y\cdot z_v=2z^2 z_u$

$z_x(z^2+2z^3z_u+y\,z_v)=2z^2z_u$

$$\boxed{z_x=\frac{2z^2z_u}{z^2+2z^3z_u+y\,z_v}}$$

$u_x=2-2z\,z_x=2-2z\cdot\frac{2z^2z_u}{z^2+2z^3z_u+y\,z_v}=\frac{2z^2+\cancel{4z^3z_u}+2y\,z_v-\cancel{4z^3z_u}}{z^2+2z^3z_u+y\,z_v}$

$$\boxed{u_x=\frac{2z^2+2y\,z_v}{z^2+2z^3z_u+y\,z_v}}$$

$v_x=-\frac{y}{z^2}\cdot z_x=-\frac{y}{\cancel{z^2}}\cdot\frac{2\cancel{z^2}z_u}{z^2+2z^3z_u+y\,z_v}=-\frac{2y\,z_u}{z^2+2z^3z_u+y\,z_v}$

$$\boxed{v_x=-\frac{2y\,z_u}{z^2+2z^3z_u+y\,z_v}}$$

$z_y=z_u u_y+z_v v_y=z_u\cdot(-2z\cdot z_y)+z_v\cdot\frac{z-y\,z_y}{z^2}$

$z_y=-2z\cdot z_u\cdot z_y+\frac{z\,z_v-y\cdot z_v\cdot z_y}{z^2}$

$z_y\cdot z^2=-2z^3z_u\,z_y+z\,z_v-y\cdot z_v\,z_y$

$z_y\cdot z^2+2z^3z_u\,z_y+y\cdot z_v\,z_y=z\cdot z_v$

$z_y(z^2+2z^3z_u+y\,z_v)=z\cdot z_v$

$$\boxed{z_y=\frac{z\cdot z_v}{z^2+2z^3z_u+y\cdot z_v}}$$

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$u_y=-2z\,z_y=-2z\cdot\frac{z\cdot z_v}{z^2+2z^3z_u+y\,z_v}=\boxed{-\frac{2z^2z_v}{z^2+2z^3z_u+y\,z_v}}$

* (33)

исправка

$A = xz\cdot z_x + yz\cdot z_y - x$

$A = xz\cdot\dfrac{2z^2 z_u}{z^2+2z^3z_u+yz_M} + yz\cdot\dfrac{z\cdot z_M}{z^2+2z^3z_u+yz_M} - x$

$A = \dfrac{\cancel{2xz^3z_u} + yz^2z_M - xz^2 - \cancel{2xz^3z_u} - xyz_M}{z^2+2z^3z_u+yz_M}$

$$\boxed{A = \dfrac{(yz^2-xy)z_M - xz^2}{z^2+2z^3z_u+yz_M}}$$

*

(33) $z = z(x,y)$; $x = z\,f\left(\dfrac{y}{z}\right)$ $\qquad x\cdot z_x + y z_y - z = ?$

$u = \dfrac{y}{z}$ $\qquad \boxed{x = z\cdot f(u)}$

$u_x = -\dfrac{y}{z^2}\cdot z_x$

$u_y = \dfrac{z - y z_y}{z^2}$

$x = z\cdot f(u) \;/\dfrac{\partial}{\partial x}$

$1 = z_x\cdot f(u) + z(f_u u_x)$

$1 = z_x\cdot f(u) + z f_u\cdot\left(-\dfrac{y\cdot z_x}{z^2}\right)$

$z^2 = z^2\cdot z_x\cdot f(u) - zy\cdot f_u\cdot z_x$

$z^2 = z\cdot z_x\,\underbrace{(z\cdot f(u))}_{x} - zy\cdot f_u\cdot z_x$

$z^2 = xz\cdot z_x - zy\cdot f_u\cdot z_x$

$z^2 = z_x\,(xz - zy f_u)$

$z_x = \dfrac{z^2}{z(x-yf_u)} = \dfrac{z}{x-yf_u}$

$$\boxed{z_x = \dfrac{z}{x-yf_u}}$$

$x = z\cdot f(u) \;/\dfrac{\partial}{\partial y}$

$0 = z_y\cdot f(u) + z\cdot f_u u_y$

$0 = z_y f(u) + z\cdot f_u\cdot\dfrac{z-yz_y}{z^2} \quad /z^2$

$z_y\cdot z\,(z f(u)) + z f_u(z - y z_y) = 0$

$x\cdot z\cdot z_y + z^2 f_u - y\cdot z f_u\cdot z_y = 0$

$z_y(xz - yzf_u) = -z^2 f_u$

$z_y = \dfrac{-z^2 f_u}{z(x-yf_u)} = -\dfrac{z f_u}{x-yf_u}$

$$\boxed{z_y = -\dfrac{z f_u}{x-yf_u}}$$

ИСПРАВКА

$A = x\cdot z_x + y z_y - z$

$A = x\cdot\dfrac{z}{x-yf_u} - y\cdot\dfrac{zf_u}{x-yf_u} - z$

$= \dfrac{\cancel{xz} - \cancel{yzf_u} - \cancel{xz} + \cancel{yzf_u}}{x-yf_u}$

$$\boxed{A = 0}$$

(14) $z = z(x,y)\,;\quad z = \varphi(x+y)\psi(x-y) \qquad z\cdot z_{xx} - z_x^2 - z z_{yy} + z_y^2 = \,?$

$u = x+y \qquad M = x-y$

$u_x = 1 \qquad M_x = 1$

$u_y = 1 \qquad M_y = -1$

$\boxed{z = \varphi(u)\psi(M)}$

$z_x = \varphi_u u_x\cdot\psi(M) + \varphi(u)\cdot\psi_M M_x = \boxed{\varphi_u\psi(M) + \varphi(u)\cdot\psi_M}$

$z_{xx} = (\varphi_{uu}u_x + \varphi_{uM}M_x)\psi(M) + \varphi_u\cdot\psi_M M_x + \varphi_u u_x\psi_M + \varphi(u)(\psi_{MM}M_x + \psi_{Mu}u_x)$

$= (\varphi_{uu} + \varphi_{uM})\psi(M) + \varphi_u\psi_M + \varphi_u\psi_M + \varphi(u)(\psi_{MM} + \psi_{Mu})$

$= \boxed{(\varphi_{uu} + \varphi_{uM})\psi(M) + 2\varphi_u\psi_M + \varphi(u)(\psi_{MM} + \psi_{Mu})}$

$z_y = \varphi_u u_y\cdot\psi(M) + \varphi(u)\cdot\psi_M M_y = \boxed{\varphi_u\psi(M) - \varphi(u)\psi_M}$

$z_{yy} = (\varphi_{uu}u_y + \varphi_{uM}M_y)\psi(M) + \varphi_u\psi_M M_y - \varphi_u u_y\psi_M - \varphi(u)(\psi_{MM}M_y + \psi_{Mu}u_y)$

$= (\varphi_{uu} - \varphi_{uM})\psi(M) - \varphi_u\psi_M - \varphi_u\psi_M + \varphi(u)(\psi_{Mu} - \psi_{MM})$

$= \boxed{(\varphi_{uu} - \varphi_{uM})\psi(M) - 2\varphi_u\psi_M - \varphi(u)(\psi_{Mu} - \psi_{MM})}$

ИСПРАВКА

$A = z\cdot z_{xx} - z_x^2 - z z_{yy} + z_y^2$

$A = z(\cancel{\varphi_{uu} + \varphi_{uM}})\psi(M) + 2z\varphi_u\psi_M + z\cdot\varphi(u)(\psi_{MM} + \psi_{Mu}) -$

$- z(\cancel{\varphi_{uu} - \varphi_{uM})\psi(M)} + 2z\varphi_u\psi_M + z\varphi(u)(\psi_{Mu} - \psi_{MM}) -$

$- (\varphi_u\psi(M) + \varphi(u)\psi_M)^2 + (\varphi_u\psi(M) - \varphi(u)\psi_M)^2$

$A = z\varphi(u)(\cancel{\psi_{MM}} + \psi_{Mu} + \psi_{Mu} - \cancel{\psi_{MM}}) - \cancel{(\varphi_u\psi(M))^2} - 2\varphi_u\psi(M)\varphi(u)\psi_M - \cancel{(\varphi(u)\psi_M)^2}$

$+ \cancel{(\varphi_u\psi(M))^2} - 2\varphi_u\psi(M)\varphi(u)\psi_M + \cancel{(\varphi(u)\psi_M)^2} + 4z\varphi_u\psi_M$

$A = z\varphi(u)\cdot 2\psi_{Mu} - 4\varphi_u\underbrace{\psi(M)\varphi(u)}_{z}\psi_M + 4z\varphi_u\psi_M$

$\boxed{A = 2z\cdot\varphi(u)\cdot\psi_{Mu} - 4z\varphi_u\psi_M + 4z\varphi_u\psi_M}$?

$\boxed{A = 2z\varphi(u)\psi_{Mu}}$

(34)

(35)

34. $z=z(x,y)$; $\frac{x}{z}-1=\ln\frac{z}{y}$ $\quad z_x+z_y=?$

$$\frac{x}{z}-1=\ln\left(\frac{z}{y}\right) \Big/ \frac{\partial}{\partial x}$$

$$\frac{z-x\cdot z_x}{z^2}=\frac{1}{\frac{z}{y}}\cdot\frac{z_x}{y}$$

$$\frac{z-xz_x}{z^2}=\frac{\cancel{y}}{z}\cdot\frac{z_x}{\cancel{y}}$$

$$\frac{z-xz_x}{z^{\cancel{2}}}=\frac{z_x}{\cancel{z}}$$

$$z\cdot z_x=z-xz_x$$

$$z\cdot z_x+xz_x=z$$

$$z_x(z+x)=z$$

$$\boxed{z_x=\frac{z}{x+z}}$$

$$\frac{x}{z}-1=\ln\frac{z}{y} \Big/ \frac{\partial}{\partial y}$$

$$-\frac{x}{z^2}\cdot z_y=\frac{1}{\frac{z}{y}}\cdot\frac{z_y\cdot y-z}{y^2}$$

$$\frac{-x\cdot z_y}{z^2}=\frac{y}{z}\cdot\frac{z_y\cdot y-z}{y^2}$$

$$\frac{-x\cdot z_y}{z^2}=\frac{z_y\cdot y-z}{z\cdot y}$$

$$yz\cdot z_y-z^2=-xyz_y$$

$$yzz_y+xyz_y=z^2$$

$$z_y(xy+yz)=z^2$$

$$\boxed{z_y=\frac{z^2}{y(x+z)}}$$

$$A=z_x+z_y=\frac{z}{x+z}+\frac{z^2}{y(x+z)}=\frac{zy+z^2}{y(x+z)}=\boxed{\frac{z(y+z)}{y(x+z)}}$$

35. $z=z(x,y)$; $(x^2+y^2+z^2)^3=y^2z$ $\quad A=(x^2-y^2-z^2)z_x+2xy\,z_y-2xz$

$$(x^2+y^2+z^2)^3=y^2z \Big/ \frac{\partial}{\partial x}$$

$$3(x^2+y^2+z^2)^2(2x+2z\cdot z_x)=y^2\cdot z_x$$

$$6x(x^2+y^2+z^2)^2+6z\cdot z_x(x^2+y^2+z^2)^2=y^2z_x$$

$$6z\cdot z_x(x^2+y^2+z^2)^2-y^2\cdot z_x=6x(x^2+y^2+z^2)^2$$

$$z_x\left(6z(x^2+y^2+z^2)^2-y^2\right)=6x(x^2+y^2+z^2)^2$$

$$\boxed{z_x=\frac{6x(x^2+y^2+z^2)^2}{6z(x^2+y^2+z^2)^2-y^2}}$$

$$(x^2+y^2+z^2)^3=y^2z \Big/ \frac{\partial}{\partial y}$$

$$3(x^2+y^2+z^2)^2(2y+2z\cdot z_y)=2y\cdot z+y^2\cdot z_y$$

$$6y(x^2+y^2+z^2)^2+6z\cdot z_y(x^2+y^2+z^2)^2=2y\cdot z+y^2z_y$$

$$6z\cdot z_y(x^2+y^2+z^2)^2-y^2z_y=2yz-6y(x^2+y^2+z^2)^2$$

$$z_y\left(6z(x^2+y^2+z^2)^2-y^2\right)=2yz-6y(x^2+y^2+z^2)^2$$

$$\boxed{z_y=\frac{2yz-6y(x^2+y^2+z^2)^2}{6z(x^2+y^2+z^2)^2-y^2}}$$

$$A=(x^2+y^2+z^2)z_x+2xy\,z_y-2xz$$

$$A=(x^2+y^2+z^2)\frac{6x(x^2+y^2+z^2)^2}{6z(x^2+y^2+z^2)^2-y^2}+2xy\cdot\frac{2yz-6y(x^2+y^2+z^2)^2}{6z(x^2+y^2+z^2)^2-y^2}-2xz$$

$$A=\frac{6x(x^2+y^2+z^2)^3+4xy^2z-12xy^2(x^2+y^2+z^2)^2-2xz(6z(x^2+y^2+z^2)^2-y^2)}{6z(x^2+y^2+z^2)^2-y^2}$$

$$A=\frac{6x(x^2+y^2+z^2)^3+4xy^2z-12xy^2(x^2+y^2+z^2)^2-12xz^2(x^2+y^2+z^2)^2+2xy^2z}{6z(x^2+y^2+z^2)^2-y^2}$$

$$A=\frac{6x(x^2+y^2+z^2)^3+6xy^2z-12x(x^2+y^2+z^2)^2(y^2+z^2)}{6z(x^2+y^2+z^2)^2-y^2}$$

$$A=\frac{6x(x^2+y^2+z^2)^2(x^2+y^2+z^2-2y^2-2z^2)+6xy^2z}{6z(x^2+y^2+z^2)^2-y^2}$$

$$\boxed{A=\frac{6x(x^2+y^2+z^2)^2(x^2-y^2-z^2)+6xy^2z}{6z(x^2+y^2+z^2)^2-y^2}}$$

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36. $z=z(x,y)$; $z=x\varphi\left(\frac{x}{y^2}\right)$ $\quad A=2xz_x+yz_y-2z$

$u=\frac{x}{y^2}\quad u_x=\frac{1}{y^2}\quad u_y=-\frac{2x}{y^3}\qquad z_x=\varphi_u u_x$

$z=x\cdot\varphi(u)$

$z_x=\varphi(u)+x\cdot\varphi_u u_x=\varphi(u)+x\cdot\varphi_u\cdot\frac{1}{y^2}$

$z_y=x\cdot\varphi_u u_y=x\cdot\varphi_u\cdot\left(-\frac{2x}{y^3}\right)=-\frac{2x^2}{y^3}\varphi_u$

ИСПРАВКА

$A=2xz_x+yz_y-2z$

$A=2x\cdot\left(\varphi(u)+x\cdot\varphi_u\cdot\frac{1}{y^2}\right)-y\cdot\frac{2x^2}{y^3}\varphi_u-2(x\cdot\varphi(u))$

$A=\cancel{2x\varphi(u)}+2\frac{x^2}{y^2}\varphi_u-2\frac{x^2}{y^2}\varphi_u-\cancel{2x\varphi(u)}=0$

$\boxed{A=0}$

39.

ИСПРАВКА

37. $z=z(x,y)$; $z=x\cdot\varphi(\sqrt{x^2+y^2})$ $\qquad A=y\cdot z_x-x\cdot z_y$

$u=\sqrt{x^2+y^2}$

$u_x=\frac{1}{2\sqrt{x^2+y^2}}\cdot 2x=\frac{x}{\sqrt{x^2+y^2}}$

$u_y=\frac{1}{2\sqrt{x^2+y^2}}\cdot 2y=\frac{y}{\sqrt{x^2+y^2}}$

$z=x\cdot\varphi(u)\Rightarrow\varphi(u)=\frac{z}{x}$

$z_x=\varphi(u)+x\cdot\varphi_u u_x=\varphi(u)+x\cdot\varphi_u\cdot\frac{x}{\sqrt{x^2+y^2}}=\varphi(u)+\frac{x^2}{\sqrt{x^2+y^2}}\varphi_u$

ИСПРАВКА

40.

ИСПРАВКА

$$z_y = x\cdot\varphi_u u_y = x\cdot\varphi_u\cdot\frac{y}{\sqrt{x^2+y^2}} = \frac{xy}{\sqrt{x^2+y^2}}\varphi_u$$

$$A = y\cdot z_x - x\cdot z_y$$

$$A = y\cdot\left(\varphi(u)+\frac{x^2}{\sqrt{x^2+y^2}}\varphi_u\right) - x\cdot\frac{xy}{\sqrt{x^2+y^2}}\varphi_u$$

$$A = y\cdot\varphi(u) + \cancel{\frac{x^2y}{\sqrt{x^2+y^2}}\varphi_u} - \cancel{\frac{x^2y}{\sqrt{x^2+y^2}}\varphi_u}$$

$$A = y\cdot\varphi(u) = y\cdot\frac{z}{x} = \boxed{\frac{yz}{x}}$$

38. $z = z(x,y)$; $z = x^2\cdot\varphi\left(\frac{y}{x^2}\right)$ $\qquad A = xz_x + 2yz_y - 2z$

$$u = \frac{y}{x^2}$$

$$u_x = -\frac{2y}{x^3} \qquad u_y = \frac{1}{x^2}$$

$$z = x^2\cdot\varphi(u)$$

$$z_x = 2x\cdot\varphi(u) + x^2\cdot\varphi_u u_x = 2x\varphi(u) - \cancel{x^2}\frac{2y}{x^{\cancel{3}}}\varphi_u = 2x\varphi(u) - \frac{2y}{x}\varphi_u$$

$$z_y = x^2\cdot\varphi_u u_y = \cancel{x^2}\cdot\frac{1}{\cancel{x^2}}\varphi_u = \varphi_u$$

$$A = x\cdot z_x + 2y\cdot z_y - 2z$$

$$A = x\left(2x\varphi(u) - \frac{2y}{x}\varphi_u\right) + 2y\,\varphi_u - 2z$$

$$A = \cancel{2x^2\varphi(u)} - \cancel{2y\varphi_u} + \cancel{2y\varphi_u} - \cancel{2x^2\varphi(u)} = 0$$

$$\boxed{A = 0}$$

ИСПРАВКА

39. $z = z(x,y)$; $z = \varphi(x)\psi(y)$ $\qquad A = z_x z_y - z\cdot z_{xy}$

$$z_x = \varphi_x$$

$$z_{xy} = 0$$

?

$$z_y = \psi_y$$

$$A = z_x z_y - z\cdot z_{xy}$$

$$\boxed{A = \varphi_x\psi_y}$$

ИСПРАВКА

40. $z = z(x,y)$; $z = x\,\varphi(x+y) + y\cdot\psi(x+y)$ $\qquad A = z_{xx} - 2z_{xy} + z_{yy}$

$$u = x+y \quad u_x = 1 \quad u_y = 1$$

$$z = x\cdot\varphi(u) + y\,\psi(u)$$

$$z_x = \varphi(u) + x\cdot\varphi_u u_x + y\cdot\psi_u u_x = \varphi(u) + x\cdot\varphi_u + y\,\psi_u$$

$$z_{xx} = \varphi_u u_x + \varphi_u + x(\varphi_{uu}u_x) + y\,\psi_{uu}u_x = 2\varphi_u + x\varphi_{uu} + y\,\psi_{uu}$$

ИСПРАВКА

$z = x\varphi(u) + y\psi(u)$

$z_y = x\cdot\varphi_u u_y + \psi(u) + y\psi_u u_y = x\cdot\varphi_u + \psi(u) + y\cdot\psi_u$

$z_{yy} = x\cdot\varphi_{uu}u_y + \psi_u u_y + \psi_u + y\cdot\psi_{uu}u_y = x\varphi_{uu} + 2\psi_u + y\psi_{uu}$

$z_{yx} = \varphi_u + x\cdot\varphi_{uu}u_x + \psi_u u_x + y\cdot\psi_{uu}u_x = \varphi_u + \psi_u + x\cdot\varphi_{uu} + y\cdot\psi_{uu}$

$A = z_{xx} - 2z_{xy} + z_{yy}$

$A = \cancel{2\varphi_u} + \cancel{x\varphi_{uu}} + \cancel{y\psi_{uu}} - \cancel{2\varphi_u} - \cancel{2\psi_u} - \cancel{2x\varphi_{uu}} - \cancel{2y\psi_{uu}} + \cancel{x\varphi_{uu}} + \cancel{2\psi_u} + \cancel{y\psi_{uu}}$

$\boxed{A = 0}$

(41.) $z = z(x,y)$; $z = \varphi\left(\frac{y}{x}\right) + x\cdot\psi\left(\frac{y}{x}\right)$ $\quad A = x^2 z_{xx} + 2xy z_{xy} + y^2 z_{yy}$

$u = \frac{y}{x} \quad u_x = -\frac{y}{x^2} \quad u_y = \frac{1}{x}$

$z = \varphi(u) + x\cdot\psi(u)$

$z_x = \varphi_u u_x + \psi(u) + x\cdot\psi_u u_x = \varphi_u\left(-\frac{y}{x^2}\right) + \psi(u) + x\cdot\frac{1}{x}\psi_u \Rightarrow$

$\boxed{z_x = -\frac{y}{x^2}\varphi_u + \psi(u) + \psi_u}$

$z_{xx} = \frac{2y}{x^3}\varphi_u - \frac{y}{x^2}\varphi_{uu}u_x + \psi_u u_x + \psi_{uu}u_x = \boxed{\frac{2y}{x^3}\varphi_u + \frac{y^2}{x^4}\varphi_{uu} - \frac{y}{x^2}\psi_u - \frac{y}{x^2}\psi_{uu}}$

$z_{xy} = -\frac{1}{x^2}\varphi_u - \frac{y}{x^2}\varphi_{uu}u_y + \psi_u u_y + \psi_{uu}u_y = \boxed{-\frac{1}{x^2}\varphi_u - \frac{y}{x^3}\varphi_{uu} + \frac{1}{x}\psi_u + \frac{1}{x}\psi_{uu}}$

$z_y = \varphi_u u_y + x\psi_u u_y = \boxed{\frac{1}{x}\varphi_u + \psi_u}$

$z_{yy} = \frac{1}{x}\varphi_{uu}u_y + \psi_{uu}u_y = \boxed{\frac{1}{x^2}\varphi_{uu} + \frac{1}{x}\psi_{uu}}$

$A = x^2 z_{xx} + 2xy z_{xy} + y^2 z_{yy}$

$A = x^2\left(\frac{2y}{x^3}\varphi_u + \frac{y^2}{x^4}\varphi_{uu} - \frac{y}{x^2}\psi_u - \frac{y}{x^2}\psi_{uu}\right) + 2xy\left(-\frac{1}{x^2}\varphi_u - \frac{y}{x^3}\varphi_{uu} + \frac{1}{x}\psi_u + \frac{1}{x}\psi_{uu}\right) + y^2\left(\frac{1}{x^2}\varphi_{uu} + \frac{1}{x}\psi_{uu}\right)$

$A = \cancel{\frac{2y}{x}\varphi_u} + \cancel{\frac{y^2}{x^2}\varphi_{uu}} - y\psi_u - y\psi_{uu} - \cancel{\frac{2y}{x}\varphi_u} - \cancel{\frac{2y^2}{x^2}\varphi_{uu}} + 2y\psi_u + 2y\psi_{uu} + \cancel{\frac{y^2}{x^2}\varphi_{uu}} + \frac{y^2}{x}\psi_{uu}$

$\boxed{A = y\psi_u + \left(y + \frac{y^2}{x}\right)\psi_{uu}}$

(42.) $z = \sqrt{\frac{x}{y}}\,f(xy) + g\left(\frac{x}{y}\right)$ $\quad A = x^2 z_{xx} - y^2 z_{yy} - 2y z_y$

$u = \frac{x}{y} \qquad M = xy$

$u_x = \frac{1}{y} \qquad M_x = y$

$u_y = -\frac{x}{y^2} \qquad M_y = x$

$$z = \sqrt{u}\cdot f(M) + g(u)$$

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$$z_x = \frac{1}{2\sqrt{u}}\cdot u_x \cdot f(M) + \sqrt{u}\cdot f_M M_x + g_u u_x =$$

$$= \frac{1}{2\sqrt{u}}\cdot\frac{1}{y}\, f(M) + \sqrt{u}\cdot f_M\, y + g_u\cdot\frac{1}{y}$$

$$= \frac{f(M)}{2y\sqrt{u}} + y\sqrt{u}\cdot f_M + \frac{1}{y}\cdot g_u$$

$$z_{xx} = \frac{f_M M_x\cdot 2y\sqrt{u} - f(M)\, 2y\, u_x}{4y\, u} + y\cdot\frac{1}{2\sqrt{u}}\cdot u_x\cdot f_M + y\sqrt{u}\cdot f_{MM} M_x + \frac{1}{y}\, g_{uu} u_x =$$

$$= \frac{f_M\cdot 2y^2\sqrt{u} - 2\cdot f(M)}{4x} + \frac{f_M}{2\sqrt{u}} + y^2\sqrt{u} f_{MM} + \frac{1}{y^2} g_{uu}$$

$$z_y = \frac{1}{2\sqrt{u}}\, u_y\cdot f(M) + \sqrt{u}\cdot f_M M_y + g_u u_y =$$

$$= -\frac{x\cdot f(M)}{2y^2\sqrt{u}} + x\sqrt{u}\, f_M - \frac{x}{y^2} g_u$$

$$z_{yy} = \frac{-x f_M M_y\, 2y^2\sqrt{u} + x f(M)\left(4y\sqrt{u} + \cancel{2}y^2\frac{1}{2\sqrt{u}}\, u_y\right)}{4y^4 u} + \frac{x}{2\sqrt{u}}\, u_y\, f_M + x\sqrt{u}\, f_{MM} M_y +$$

$$+ \frac{2x}{y^3} g_u - \frac{x}{y^2} g_{uu} u_y$$

$$z_{yy} = \frac{-2x^2y^2 f_M\sqrt{u} + 4xy\sqrt{u}\, f(M) - \frac{x^2 f(M)}{\sqrt{u}}}{4xy^3} - \frac{x}{2y^2\sqrt{u}} f_M + x^2\sqrt{u} f_{MM} + \frac{2x}{y^3} g_u - \frac{x^2}{y^4} g_{uu}$$

$$A = x^2\left(\frac{f_M\cdot 2y^2\sqrt{u} - 2f(M)}{4x} + \frac{f_M}{2\sqrt{u}} + y^2\sqrt{u} f_{MM} + \frac{1}{y^2} g_{uu}\right) -$$

$$- y^2\left(\frac{-2x^2y^2 f_M\sqrt{u} + 4xy\sqrt{u}\, f(M) - \frac{x^2 f(M)}{\sqrt{u}}}{4xy^3} - \frac{x}{2y^2\sqrt{u}} f_M + x^2\sqrt{u} f_{MM} + \frac{2x}{y^3} - \frac{x^2}{y^4} g_{uu}\right) -$$

$$- 2y\left(-\frac{x f(M)}{2y^2\sqrt{u}} + x\sqrt{u}\, f_M - \frac{x}{y^2} g_u\right)$$

$\psi_{uu})$

$$x^{\cancel{2}}\left(-\frac{2f(M)}{{}_2\cancel{4x}}\right) - y^2\left(\frac{4xy\sqrt{u}\, f(M)}{4xy^3} - \frac{x^2 f(M)}{4xy^3\sqrt{u}}\right) - 2y\left(-\frac{x f(M)}{2y^2\sqrt{u}}\right)$$

$$= -\frac{x}{2} f(M) - \sqrt{u} f(M) + \frac{x f(M)}{4y\sqrt{u}} + \frac{\cancel{2}x f(M)}{\cancel{2}y\sqrt{u}}$$

$$f(M)\left(-\frac{x}{2} - \sqrt{u} + \frac{x}{4y\sqrt{u}} + \frac{x}{y\sqrt{y}}\right)$$

$$\frac{-2xy\sqrt{u} - 4y\cdot\frac{x}{y} + x + 4x}{4y\sqrt{u}} = \frac{-2xy\sqrt{u} - 4x + 5x}{4y\sqrt{u}} = \frac{-2xy\sqrt{u} + x}{4y\sqrt{u}}$$

$$\frac{x(1 - 2y\sqrt{u})}{4y\sqrt{u}}$$

(13) $z=z(x,y)$ $\quad x^2+y^2+z^2=yg\left(\frac{y}{z}\right)$ $\quad (x^2-y^2-z^2)z_x+2xyz_y-2xz=?$

$u=\frac{y}{z}$ $\quad u_x=-\frac{y\cdot z_x}{z^2}$ $\quad u_y=\frac{z-yz_y}{z^2}$

$x^2+y^2+z^2=y\cdot g(u)\ \Big/\frac{\partial}{\partial x}$

$2x+2z\cdot z_x=y\cdot g_u u_x$

$2x+2zz_x=y\cdot g_u\cdot\left(-\frac{y\cdot z_x}{z^2}\right)$

$2xz^2-2z^3z_x=-y^2g_u\cdot z_x$

$y^2g_uz_x+2z^3z_x=-2xz^2$

$z_x(y^2g_u-2z^3)=-2xz^2$

$$\boxed{z_x=\frac{2xz^2}{2z^3-y^2g_u}}$$

$x^2+y^2+z^2=g(u)\cdot y\ \Big/\frac{\partial}{\partial y}$

$2y+2zz_y=g_uu_y\cdot y+g(u)$

$2y+2z\cdot z_y=y\cdot g_u\frac{z-yz_y}{z^2}+\frac{x^2+y^2+z^2}{y}$

$2z\cdot z_y-\frac{yg_u(z-yz_y)}{z^2}=\frac{x^2+y^2+z^2-2y^2}{y}\ \Big/yz^2$

$2y\cdot z^3\cdot z_y-y^2g_u(z-yz_y)=z^2(x^2-y^2+z^2)$

$2yz^3z_y+y^3g_uz_y=y^2zg_u+z^2(x^2-y^2+z^2)$

$z_y(2yz^3+y^3g_u)=y^2zg_u+z^2(x^2-y^2+z^2)$

$z_y=\frac{y^2zg_u+z^2(x^2-y^2+z^2)}{y(2z^3+y^2g_u)}$

$A=\frac{2xz^2(x^2-y^2-z^2)}{2z^3-y^2g_u}+\frac{2xy(y^2zg_u+z^2(x^2-y^2+z^2))}{y(2z^3+y^2g_u)}-2xz$

$A=\frac{2xz^2(x^2-y^2-z^2)}{2z^3-y^2g_u}$

(13.) $x^2+y^2+z^2=yg(u)$ $\quad u=\frac{y}{z}$ $\quad u_x=-\frac{yz_x}{z^2}$ $\quad u_y=\frac{z-yz_y}{z^2}$

$\frac{x^2+y^2+z^2}{y}=g(u)\ \Big/\frac{\partial}{\partial x}$

$\frac{2x+2zz_x}{y}=g_u\cdot\left(-\frac{yz_x}{z^2}\right)$

$2xz^2+2z^3\cdot z_x=-y^2g_uz_x$

$2z^3z_x+y^2g_uz_x=-2xz^2$

$z_x(2z^3+y^2g_u)=-2xz^2$

$$\boxed{z_x=-\frac{2xz^2}{2z^3+y^2g_u}}$$

$\frac{x^2+y^2+z^2}{y}=g(u)\ \Big/\frac{\partial}{\partial y}$

$\frac{(2y+2zz_y)y-(x^2+y^2+z^2)}{y^2}=g_u\cdot\frac{z-yz_y}{z^2}$

$z^2(2y^2+2yzz_y-x^2-y^2-z^2)=(g_uz-yg_uz_y)y^2$

$y^2z^2+2yz^3z_y-x^2z^2-z^4=y^2zg_u-y^3g_uz_y$

$2yz^3z_y+y^3g_uz_y=y^2zg_u+z^2(x^2-y^2+z^2)$

$z_y(2yz^3+y^3g_u)=y^2zg_u+z^2(x^2-y^2+z^2)$

$$\boxed{z_y=\frac{y^2zg_u+z^2(x^2-y^2+z^2)}{y(2z^3+y^2g_u)}}$$

$A=\frac{-2xz^2(x^2-y^2-z^2)}{2z^3+y^2g_u}+2xy\cdot\frac{y^2zg_u+z^2(x^2-y^2+z^2)}{y(2z^3+y^2g_u)}-2xz$

$A=\frac{2xz(-z(x^2-y^2-z^2)+y^2g_u+z(x^2-y^2+z^2))}{2z^3+y^2g_u}-2xz$

$$A=\frac{2xz\left(z\left(\cancel{-x^2}+\cancel{y^2}+z^2+\cancel{x^2}-\cancel{y^2}+z^2\right)+y^2\ln\right)}{2z^3+y^2\ln}-2xz$$

$$A=\frac{2xz\left(\cancel{2z^3+y^2\ln}\right)}{\cancel{2z^3+y^2\ln}}-2xz=2xz-2xz=0$$

$$\boxed{A=0}$$

44. $z=z(x,y)$, $x+y+z=\ln(x^2+y^2+z^2)$ $\qquad A=(y-z)z_x+(z-x)z_y-x+y$

$$x+y+z=\ln(x^2+y^2+z^2)\ \Big/\frac{\partial}{\partial x}$$

$$1+z_x=\frac{1}{x^2+y^2+z^2}(2x+2z\,z_x)$$

$$2x+2z\,z_x=(1+z_x)(x^2+y^2+z^2)$$

$$2x+2z\,z_x=x^2+y^2+z^2+z_x(x^2+y^2+z^2)$$

$$2z\,z_x-z_x(x^2+y^2+z^2)=x^2+y^2+z^2-2x$$

$$z_x(2z-x^2-y^2-z^2)=x^2+y^2+z^2-2x$$

$$\boxed{z_x=-\frac{x^2+y^2+z^2-2x}{x^2+y^2+z^2-2z}}$$

$$x+y+z=\ln(x^2+y^2+z^2)\ \Big/\frac{\partial}{\partial y}$$

$$(1+z_y)=\frac{2y+2z\,z_y}{x^2+y^2+z^2}$$

$$2y+2z\,z_y=(1+z_y)(x^2+y^2+z^2)$$

$$\boxed{z_y=-\frac{x^2+y^2+z^2-2y}{x^2+y^2+z^2-2z}}$$

$$A=\frac{(z-y)(x^2+y^2+z^2-2x)+(x-z)(x^2+y^2+z^2-2y)}{x^2+y^2+z^2-2z}-x+y$$

$$A=\frac{\cancel{z(x^2+y^2+z^2)}-2xz-x^2y-y^3-yz^2-\cancel{z(x^2+y^2+z^2)}+2zy+x^3+xy^2+xz^2-\cancel{2xy}+\cancel{2xy}}{x^2+y^2+z^2-2z}-x+y$$

$$A=\frac{2zy-2xz-x^2y-y^3-yz^2+x^3+xy^2+xz^2}{x^2+y^2+z^2-2z}-x+y$$

$$A=\frac{2zy-2xz-y(x^2+y^2+z^2)+x(x^2+y^2+z^2)}{x^2+y^2+z^2-2z}-x+y$$

$$A=\frac{2z(y-x)+(x^2+y^2+z^2)(x-y)}{x^2+y^2+z^2-2z}-x+y$$

$$A=\frac{(x-y)\cancel{(x^2+y^2+z^2-2z)}}{\cancel{x^2+y^2+z^2-2z}}-x+y$$

$$A=x-y-x+y=0$$

$$\boxed{A=0}$$

(40.) $z = z(x,y)$; $x+y+z = xyz$ $\quad dz(2,1) = ?$

$x+y+z = xyz \;/\frac{\partial}{\partial x}$ $\quad 3+z=2z \quad \boxed{z=3}$ $\quad x+y+z = xyz \;/\frac{\partial}{\partial y}$

$1+z_x = yz + xyz_x$ $\quad 1+z_y = xz + xyz_y$

$z_x - xyz_x = yz - 1$ $\quad z_y - xyz_y = xz - 1$

$z_x(1-xy) = yz-1$ $\quad z_y(1-xy) = xz-1$

$\boxed{z_x = \frac{yz-1}{1-xy}}$ $\quad \boxed{z_y = \frac{xz-1}{1-xy}}$

$z_x(2,1) = \frac{3-1}{1-2} = 2$ $\quad z_y(2,1) = \frac{6-1}{1-2} = 5$

$dz(2,1) = \boxed{2dx + 5dy}$

уравени, али... 43

1.4 $\longrightarrow$ (14) (25) (26) ~~(42)~~ (19) $\quad$ (17, (23, 24), (36, 37, 38), 39, (40, 41),)

338 задачи

$z = f(u,v)$

$$\left.\begin{array}{l} u = u(x,y) \\ v = v(x,y) \end{array}\right\} \Rightarrow \boxed{\begin{array}{l} z_x = f_u u_x + f_v v_x \\ z_y = f_u u_y + f_v v_y \end{array}}$$

(42) $z = \sqrt{\frac{x}{y}}\, f(xy) + g\left(\frac{x}{y}\right)$ $\quad A = x^2 z_{xx} - y^2 z_{yy} - 2y z_y$

$u = \frac{x}{y} \quad u_x = \frac{1}{y} \quad u_y = -\frac{x}{y^2}$

$M = xy \quad M_x = y \quad M_y = x$

$z = \sqrt{u}\, f(M) + g(u)$

$z_x = z_u u_x + z_M M_x = \frac{1}{y} \cdot z_u + y \cdot z_M$

$z_{xx} = \frac{1}{y}\left(z_{uu} u_x + z_{uM} M_x\right) + y \cdot \left(z_{MM} M_x + z_{Mu} u_x\right) =$

$= \frac{1}{y}\left(z_{uu} \cdot \frac{1}{y} + z_{uM} \cdot y\right) + y\left(z_{MM} \cdot y + z_{Mu} \cdot \frac{1}{y}\right) =$

$= \frac{1}{y^2} z_{uu} + z_{uM} + y^2 z_{MM} + z_{Mu}$

$= \boxed{\frac{1}{y^2} z_{uu} + 2 z_{uM} + y^2 z_{MM}}$

$$z_y = z_u u_y + z_m m_y = \boxed{-\frac{x}{y^2} z_u + x \cdot z_m}$$

$$z_{yy} = \frac{2x}{y^3} z_u - \frac{x}{y^2}\left(z_{uu} u_y + z_{um} m_y\right) + x\left(z_{mm} m_y + z_{mu} u_y\right) =$$
$$= \frac{2x}{y^3} z_u - \frac{x}{y^2}\left(z_{uu}\left(-\frac{x}{y^2}\right) + z_{um} \cdot x\right) + x\left(z_{mm} \cdot x + z_{mu}\left(-\frac{x}{y^2}\right)\right) =$$
$$= \frac{2x}{y^3} z_u + \frac{x^2}{y^4} z_{uu} - \frac{x^2}{y^2} z_{um} + x^2 z_{mm} - \frac{x^2}{y^2} z_{mu} =$$
$$= \boxed{\frac{x^2}{y^4} z_{uu} - \frac{2x^2}{y^2} z_{um} + x^2 z_{mm} + \frac{2x}{y^3} z_u}$$

$$A = x^2 z_{xx} - y^2 z_{yy} - 2y z_y$$

$$A = x^2\left(\frac{1}{y^2} z_{uu} + 2 z_{um} + y^2 z_{mm}\right) - y^2\left(\frac{x^2}{y^4} z_{uu} - \frac{2x^2}{y^2} z_{um} + x^2 z_{mm} + \frac{2x}{y^3} z_u\right) - 2y\left(-\frac{x}{y^2} z_u + x z_m\right)$$

$$A = \cancel{\frac{x^2}{y^2} z_{uu}} + 2x^2 z_{um} + \cancel{x^2 y^2 z_{mm}} - \cancel{\frac{x^2}{y^2} z_{uu}} + 2x^2 z_{um} - \cancel{x^2 y^2 z_{mm}} - \cancel{\frac{2x}{y} z_u} + \cancel{\frac{2x}{y} z_u} - 2xy z_m$$

$$\boxed{A = 4x^2 z_{um} - 2xy z_m}$$ ✓

14. $z = z(x,y)$ $\quad z = \varphi(x+y)\,\psi(x-y)$ $\quad A = z \cdot z_{xx} - z_x^2 - z \cdot z_{yy} + z_y^2 = ?$

$u = x+y \quad u_x = 1 \quad u_y = 1$

$m = x-y \quad m_x = 1 \quad m_y = -1$

$$z_x = z_u u_x + z_m m_x = \boxed{z_u + z_m}$$

$$z_{xx} = z_{uu} u_x + z_{um} m_x + z_{mm} m_x + z_{mu} u_x = \boxed{z_{uu} + 2 z_{um} + z_{mm}}$$

$$z_{xy} = z_{uu} u_y + \cancel{z_{um} m_y} + z_{mm} m_y + \cancel{z_{mu} u_y} = z_{uu} - z_{mm}$$

$$z_y = z_u u_y + z_m m_y = \boxed{z_u + z_m}$$

$$z_{yy} = z_{uu} u_y + \cancel{z_{um} m_y} + z_{mm} m_y + \cancel{z_{mu} u_y} = \boxed{z_{uu} - z_{mm}}$$

$$A = z \cdot z_{xx} - z_x^2 - z \cdot z_{yy} + z_y^2$$

$$A = z\left(z_{uu} + 2z_{um} + z_{mm}\right) - \cancel{\left(z_u + z_m\right)^2} - z\left(z_{uu} - z_{mm}\right) + \cancel{\left(z_u + z_m\right)^2}$$

$$A = z\left(\cancel{z_{uu}} + 2z_{um} + z_{mm} - \cancel{z_{uu}} + z_{mm}\right)$$

$$\boxed{A = z\left(2z_{um} + 2z_{mm}\right)}$$ ✓

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+ 14 23 24 25 28 19

17. $z(x,y) = e^{y} \cdot f\left(y \cdot e^{\frac{x^2}{2y^2}}\right)$ $\quad (z_x, z_y)$

36. $z = z(x,y)$; $\quad z = x\varphi\left(\frac{x}{y^2}\right)$ $\quad (z_x, z_y)$

37. $z = z(x,y)$; $\quad z = x\varphi\left(\sqrt{x^2+y^2}\right)$ $\quad (z_x, z_y)$

38. $z = z(x,y)$; $\quad z = x^2\varphi\left(\frac{y}{x^2}\right)$ $\quad (z_x, z_y)$

39. $z = z(x,y)$; $\quad z = \varphi(x)\psi(y)$ $\quad (z_x, z_y, z_{xy})$

40. $z = z(x,y)$; $\quad z = x\varphi(x+y) + y\psi(x+y)$ $\quad (z_{xx}, z_{xy}, z_{yy})$

41. $z = z(x,y)$; $\quad z = \varphi\left(\frac{y}{x}\right) + x\psi\left(\frac{y}{x}\right)$ $\quad (z_{xx}, z_{xy}, z_{yy})$

45. $z = z(x,y)$; $\quad x^2 + y^2 + z^2 = y \cdot g\left(\frac{y}{z}\right)$ $\quad (z_x, z_y)$

1.5 Локални екстремуми ф-ја више променљивих

(1.) $z(x,y) = x^2 - xy + y^2 - 2x + y$

$z_x = 2x - y - 2$

$z_y = -x + 2y + 1$

$2x - y - 2 = 0$

$-x + 2y + 1 = 0 \quad /2$

$2x - y - 2 = 0$

$-2x + 4y + 2 = 0$

$3y = 0 \Rightarrow \boxed{y = 0}$

$x = 2y + 1$

$\boxed{x = 1} \Rightarrow \boxed{M(1,0)}$

$z_{xx} = 2$ $\quad r = z_{xx} = 2$

$z_{yy} = 2$ $\quad t = z_{yy} = 2$

$z_{xy} = -1$ $\quad s = z_{xy} = -1$

$\Delta = rt - s^2 = 4 - 1 = 3$

$\Delta > 0;\ r > 0 \rightarrow$ $\boxed{M(1,0) \rightarrow \text{лок. мин.}}$ ✓

(2.) $z(x,y) = x^3 - 3xy + y^3$

$z_x = 3x^2 - 3y$

$z_y = -3x + 3y^2$

$z_{xx} = 3x$

$z_{xy} = -3$

$z_{yy} = 6y$

$\cancel{3}x^2 - \cancel{3}y = 0$

$-\cancel{3}x + \cancel{3}y^2 = 0$

$y = x^2$

$-x + x^4 = 0$

$x(x^3 - 1) = 0 \rightarrow \boxed{x = 0} \vee \quad x^3 - 1 = 0$

$\boxed{y = 0} \quad\quad x^3 = 1$

$M_1(0,0) \quad\quad \boxed{x = 1}$

$\boxed{y = 1}$

$M_2(1,1)$

$M_1(0,0)$

$r = 0;\ t = 0;\ s = -3;\ \Delta = -9$

$\Delta < 0; \rightarrow$ $\boxed{M_1 \rightarrow \text{није екстрем}}$

$M_2(1,1)$

$r = 3;\ t = 6;\ s = -3;\ \Delta = 18 - 9 = 9$

$\Delta > 0;\ r > 0 \rightarrow$ $\boxed{M_2 \rightarrow \text{лок. мин}}$ ✓

3. $z(x,y) = 2x^3 - xy^2 + 5x^2 + y^2$

$z_x = 6x^2 - y^2 + 10x$ $\qquad$ $6x^2 - y^2 + 10x = 0$

$z_y = -2xy + 2y$ $\qquad$ $-2xy + 2y = 0$

$y^2 = 6x^2 + 10x$

$\boxed{y = \sqrt{2x(3x+5)}}$

$2y(1-x) = 0$

$2\sqrt{2x(3x+5)} \cdot (1-x) = 0$

1° $1 - x = 0$

$x = 1$

$y = \pm 4$

$\boxed{M_1(1,4)}$

$\boxed{M_2(1,-4)}$

2° $x(3x+5) = 0$

$x = 0 \quad \vee \quad 3x + 5 = 0$

$y = 0 \qquad\qquad 3x = -5$

$\boxed{M_3(0,0)} \qquad x = -\frac{5}{3}$

$y = 0$

$\boxed{M_4\left(-\frac{5}{3}, 0\right)}$

$$\boxed{\begin{array}{ll} z_{xx} = 12x + 10 & r \\ z_{yy} = -2x + 2 & t \\ z_{xy} = -2y & s \end{array}}$$

$\underline{M_1(1,4)}$

$r = 22;\ t = 0;\ s = -8;\ \Delta = -64$

$\Delta < 0 \Rightarrow \boxed{M_1 \rightarrow \text{НИЈЕ ЕКСТРЕМ}}$

$\underline{M_2(1,-4)}$

$r = 22;\ t = 0,\ s = 8;\ \Delta = -64$

$\Delta < 0 \Rightarrow \boxed{M_2 \rightarrow \text{НИЈЕ ЕКСТРЕМ}}$

$\underline{M_3(0,0)}$

$r = 10;\ t = 2;\ s = 0;\ \Delta = 20$

$\Delta > 0;\ r > 0 \rightarrow \boxed{M_3 \rightarrow \text{ЛОК. МИН.}}$

$\underline{M_4\left(-\frac{5}{3}, 0\right)}$

$r = -10;\ t = \frac{16}{3};\ s = 0;\ \Delta =$

$\Delta < 0$ $\rightarrow \boxed{M_4 \rightarrow \cancel{\text{ЛОК. МАКС.}}}$ није екстрем

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4. $z(x,y) = x^3 - 6y^2 + 6xy$

$z_x = 3x^2 + 6y$ $\qquad 3x^2 + 6y = 0$

$z_y = -12y + 6x$ $\qquad -12y + 6x = 0$ $\qquad 6 = -12y$

$x^2 + 2y = 0$ $\qquad y = -\frac{6}{12} = -\frac{1}{2}$

$x - 2y = 0$

$x + x^2 = 0$

$x(x+1) = 0$

$x = 0 \quad \vee \quad x = -1$

$y = 0 \qquad y = -\frac{1}{2}$

$\boxed{M_1(0,0)}$ $\quad$ $\boxed{M_2\left(-1, -\frac{1}{2}\right)}$

$\boxed{\begin{array}{l} z_{xx} = 6x \\ z_{yy} = -12 \\ z_{xy} = 6 \end{array}}$ $\begin{array}{l} r \\ t \\ s \end{array}$

$M_1(0,0)$

$r = 0;\ t = -12;\ s = 6;\ \Delta = -36$

$\Delta < 0 \rightarrow \boxed{M_1 \rightarrow \text{НИЈЕ ЕКСТРЕМ}}$

$M_2\left(-1, -\frac{1}{2}\right)$

$r = -6;\ t = -12;\ s = 6;\ \Delta = 72 - 36 = 36$

$\Delta > 0;\ r < 0 \Rightarrow \boxed{M_2 \rightarrow \text{ЛОК. МАКС.}}$ ✓

5. $z(x,y) = x^2y + xy^2 - 3xy - 2x - 2y$

$z_x = 2xy + y^2 - 3y - 2$ $\qquad 2xy + y^2 - 3y - 2 = 0$

$z_y = x^2 + 2xy - 3x - 2$ $\qquad x^2 + 2xy - 3x - 2 = 0$

$\cancel{2xy} + y^2 - 3y - \cancel{2} = x^2 + \cancel{2xy} - 3x - \cancel{2}$

$M_2(-0{,}46; -0{,}46)$ $\qquad y^2 - 3y = x^2 - 3x$

$r = -0{,}92;\ t = -0{,}92;\ s = -4{,}84$ $\qquad y^2 - x^2 = 3y - 3x$

$\Delta = -22{,}58$ $\qquad (y-x)(y+x) = 3(y-x)$

$\Delta < 0 \Rightarrow \boxed{M_1 \rightarrow \text{НИЈЕ ЕКСТРЕМ}}$ $\qquad (y-x)(y+x) - 3(y-x) = 0$

$M_1(1{,}46; 1{,}46)$ $\qquad (y-x)(y+x-3) = 0$

$r = 2{,}92;\ t = 2{,}92;\ s = 2{,}84$ $\qquad 1^\circ\ y - x = 0 \rightarrow y = x$

$2x^2 + x^2 - 3x - 2 = 0$

$\Delta = 0{,}46$ $\qquad 3x^2 - 3x - 2 = 0$

$\Delta > 0;\ r > 0 \Rightarrow \boxed{M_2 \rightarrow \text{ЛОК. МИН.}}$ $\qquad x_{1/2} = \frac{3 \pm \sqrt{9+24}}{6} = \frac{3 \pm \sqrt{33}}{6}$

$\boxed{M_1\left(\frac{3+\sqrt{33}}{6}, \frac{3+\sqrt{33}}{6}\right)}$ $\quad$ $\boxed{M_2\left(\frac{3-\sqrt{33}}{6}, \frac{3-\sqrt{33}}{6}\right)}$

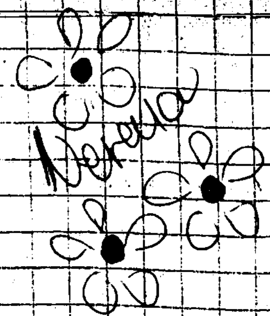
2° $y+x-3=0 \rightarrow y=3-x$

$x^2+2x(3-x)-3x-2=0$

$x^2+6x-2x^2-3x-2=0$

$-x^2+3x-2=0$

$x_{1/2}=\dfrac{-3\pm\sqrt{9-8}}{-2}=\dfrac{-3\pm1}{-2}=\begin{cases}2 \rightarrow y=1 \\ 1 \rightarrow y=2\end{cases}$ $\begin{matrix} M_3(2,1) \\ M_4(1,2)\end{matrix}$



$$\boxed{\begin{aligned} z_{xx}&=2y & r\\ z_{yy}&=2x & t\\ z_{xy}&=2x+2y-3 & s\end{aligned}}$$

$\underline{M_1(1,2)}$

$\downarrow$

$r=4;\ t=2;\ s=3;\ \Delta=8-9=-1$

$\Delta<0 \Rightarrow$ $\boxed{M_1 \rightarrow \text{није екстрем}}$

$\underline{M_2(2,1)}$

$\downarrow$

$r=2;\ t=4;\ s=3;\ \Delta=8-9=-1$

$\Delta<0 \rightarrow$ $\boxed{M_2 \rightarrow \text{није екстрем}}$

(6.) $z(x,y)=(x-y)^3+x^4+y^4$

$z_x=3(x-y)^2+4x^3$

$z_y=-3(x-y)^2+4y^3$

$3(x-y)^2+4x^3=0$

$-3(x-y)^2+4y^3=0$

$4x^3+4y^3=0$

$x^3=-y^3$

$\boxed{x=-y}$

$3(-y-y)^2+4(-y)^3=0$

$3\cdot4y^2-4y^3=0$

$4y^2(3-y)=0$

$y^2=0 \quad \vee \quad y=3$

$\Downarrow \qquad\qquad x=-3$

$y=0 \qquad \boxed{M_2(-3,3)}$

$x=0$

$\boxed{M_1(0,0)}$

$r=-36+108=72$

$t=-36+108=72$

$s=36$

$\Delta x^3+\Delta y^3=(\Delta x+\Delta y)(\Delta x^2-\Delta x\Delta y+\Delta y^2)$

$$\boxed{\begin{aligned} z_{xx}&=6(x-y)+12x^2 & r\\ z_{yy}&=6(x-y)+12y^2 & t\\ z_{xy}&=-6(x-y) & s\end{aligned}}$$

$\underline{M_1(0,0)}$

$\downarrow$

$r=0;\ t=0;\ s=0;\ \Delta=0$

$\Delta=0 \rightarrow$ потребна додатна испитивања

$\Delta z(0,0)=z(0+\Delta x,0+\Delta y)-z(0,0)$

прираштај

$\Delta z(0,0)=(\Delta x-\Delta y)^3+\Delta x^4+\Delta y^4$ $\begin{cases}\Delta x=\Delta y;\ \Delta x>0,\ \Delta y>0 \rightarrow \Delta z>0\\ \Delta x=\Delta y;\ \Delta x<0,\ \Delta y<0 \rightarrow \Delta z>0\end{cases}$

$\Delta z(0,0)\gtrless 0 \rightarrow$ $\boxed{M_1 \rightarrow \text{лок. мин. } ?}$

$\underline{M_2(-3,3)}$
$\downarrow$
$r=72$; $t=72$; $s=36$; $\Delta = 72^2-36^2=(2\cdot36)^2-36^2=4\cdot36^2-36^2=3\cdot36^2$

$\Delta>0$, $r>0 \Rightarrow \boxed{M_2 \to \text{лок. мин.}}$ ✓

(7) $z(x,y)=x^3-6xy+xy^2$

$z_x=3x^2-6y+y^2$

$z_y=-6x+2xy$

$3x^2-6y+y^2=0$
$-6x+2xy=0$

$-2x(3-y)=0$

1° $x=0$

$y^2-6y=0$

$y(y-6)=0 \quad y=0, \quad y=6$

$\Rightarrow \boxed{M_1(0,0)} \quad \boxed{M_2(0,6)}$

2° $3-y=0 \Rightarrow y=3$

$3x^2-18+9=0$
$3x^2-9=0$
$x^2-3=0$
$x^2=3 \quad x=\sqrt3, \quad x=-\sqrt3$

$\boxed{M_3(\sqrt3,3)} \quad \boxed{M_4(-\sqrt3,3)}$

$3x^2=6y-y^2$

$x^2=\frac{y(6-y)}{3}$

$x=\pm\sqrt{\frac{y(6-y)}{3}}$

1° $x_1=\sqrt{\frac{y(6-y)}{3}}$

$-6\sqrt{\frac{y(6-y)}{3}}+2y\sqrt{\frac{y(6-y)}{3}}=0$

$-6\sqrt{\frac{y(6-y)}{3}}=-2y\sqrt{\frac{y(6-y)}{3}} \quad /^2$

$9\frac{y(6-y)}{3}=y^2\cdot\frac{y(6-y)}{3}$

$3y(6-y)=\frac{y^3(6-y)}{3}$

$9y(6-y)-y^3(6-y)=0$

$(6-y)(9y-y^3)=0$

$y(6-y)(9-y^2)=0$

$y(6-y)(3-y)(3+y)=0$

$y=0 \quad x=0$
$y=6 \quad x=0$
$y=3 \quad x=\pm\sqrt3$
$y=-3 \quad x$ – не може

$$\boxed{\begin{array}{l} z_{xx}=6x \\ z_{yy}=2x \\ z_{xy}=-6+2y \end{array}}$$

$\underline{M_1(0,0)}$
$\downarrow$
$r=0$; $t=0$; $s=-6$; $\Delta=-36$

$\Delta<0 \Rightarrow \boxed{M_1 \to \text{НИЈЕ ЕКСТРЕМ}}$

$\underline{M_2(0,6)}$
$\downarrow$
$r=0$; $t=12$; $s=6$; $\Delta=-36$

$\Delta<0 \Rightarrow \boxed{M_2 \to \text{НИЈЕ ЕКСТРЕМ}}$

$\underline{M_3(\sqrt3,3)}$
$\downarrow$
$r=6\sqrt3$; $t=6$; $s=0$; $\Delta=36\sqrt3$

$\Delta>0$, $r>0 \Rightarrow \boxed{M_3 \to \text{лок. мин.}}$

$\underline{M_4(-\sqrt3,3)}$ $-6\sqrt3$
$\downarrow$
$r=-6\sqrt3$; $\cancel{t=6}$; $s=0$; $\Delta=-36\sqrt3$

$\Delta<0 \Rightarrow \boxed{\cancel{M_4 \to \text{НИЈЕ ЕКСТРЕМ}}}$ $\Delta=36$ лок. max

8. $z(x,y) = x^3 + xy^2 - y^2 + 2x^2$

$z_x = 3x^2 + y^2 + 4x$ $\qquad 3x^2 + y^2 + 4x = 0$

$z_y = 2xy + 2y$ $\qquad 2xy + 2y = 0$

$y^2 = -3x^2 - 4x$

$y = \pm\sqrt{-3x^2-4x}$ $\rightarrow$ добијам 2 решења?

$2x\sqrt{-3x^2-4x} + 2\sqrt{-3x^2-4x} = 0$

$2x\sqrt{-3x^2-4x} = (-2)\sqrt{-3x^2-4x} \quad /^2$

$x^2(-3x^2-4x) = -3x^2-4x$

$(-3x^2-4x)(x^2-1) = 0$

$x(-3x-4)(x-1)(x+1) = 0$

$x=0 \ \vee \ 3x=-4 \ \vee \ x=1 \ \vee \ x=-1$

$x=0$: $y=0$ $\downarrow$ $\boxed{M_1(0,0)}$

$3x=-4$: $x=-\frac{4}{3}$, $y=0$ $\downarrow$ $\boxed{M_2(-\frac{4}{3},0)}$

$x=1$: $\downarrow$ НЕМОГУЋЕ

$x=-1$: $y=1 \ \vee \ y=-1$ $\downarrow$ $\boxed{M_3(-1,-1)}$ $\quad \boxed{M_4(-1,1)}$

$\boxed{\begin{array}{ll} z_{xx} = 6x+4 & r \\ z_{yy} = 2x+2 & t \\ z_{xy} = 2y & s \end{array}}$

$M_1(0,0)$ $\downarrow$

$r=4;\ t=2;\ s=0;\ \Delta=8$

$\Delta>0;\ r>0 \Rightarrow$ $\boxed{M_1 \rightarrow \text{лок. мин.}}$

$M_2(-\frac{4}{3},0)$ $\downarrow$

$r=-4;\ t=-\frac{2}{3};\ s=0;\ \Delta=\frac{8}{3}$

$\Delta>0;\ r<0 \Rightarrow$ $\boxed{M_2 \rightarrow \text{лок. макс.}}$

$M_3(-1,-1)$ $\downarrow$

$r=-2;\ t=0;\ s=-2;\ \Delta=-4$

$\Delta<0 \rightarrow$ $\boxed{M_3 \rightarrow \text{није екстрем}}$

$M_4(-1,1)$ $\downarrow$

$r=-2;\ t=0;\ s=2;\ \Delta=-4$

$\Delta<0 \rightarrow$ $\boxed{M_4 \rightarrow \text{није екстрем}}$

9. $z(x,y) = x^4 + y^4 - 2(x-y)^2$

$z_x = 4x^3 - 4(x-y)$ $\qquad 4x^3 - 4(x-y) = 0$

$z_y = 4y^3 + 4(x-y)$ $\qquad 4y^3 + 4(x-y) = 0$

$4x^3 + 4y^3 = 0$

$x^3 = -y^3$

$\boxed{x=-y}$

$-4y^3 - 4(-2y) = 0$

$-4y^3 + 8y = 0$

$-y^3 + 2y = 0$

$y(2-y^2) = 0$

$y=0 \ \vee \ y^2=2$

$x=0$ $\qquad y=\pm\sqrt{2}$

$\boxed{M_1(0,0)}$ $\quad \boxed{M_2(\sqrt{2},-\sqrt{2})}$ $\quad \boxed{M_3(-\sqrt{2},\sqrt{2})}$

$$\boxed{\begin{array}{l} z_{xx} = 12x^2-4 \quad r \\ z_{yy} = 12y^2-4 \quad t \\ z_{xy} = 4 \quad s \end{array}}$$

$M_1(0,0)$

$r=-4;\ t=-4;\ s=4;\ \Delta=0$

$\Delta z(0,0) = z(0+\Delta x, 0+\Delta y) - z(0,0)$

$\Delta z(0,0) = \Delta x^4 + \Delta y^4 - 2(\Delta x - \Delta y)^2 \Rightarrow \begin{cases} \Delta x = \Delta y;\ \Delta x > 0;\ \Delta z > 0 \\ \Delta x = \Delta y;\ \Delta x < 0;\ \Delta z > 0 \end{cases}$

$\Rightarrow \Delta z(0,0) > 0 \Rightarrow \boxed{M_1 \rightarrow \text{лок. мин.}}$

$M_2(\sqrt{2}, -\sqrt{2})$

$r=20;\ t=20;\ s=4;\ \Delta = 400-16$

$\Delta > 0;\ r > 0 \Rightarrow \boxed{M_2 \rightarrow \text{лок. мин.}}$ ✓ Прираштај

$M_3(-\sqrt{2}, \sqrt{2})$

$r=20;\ t=20;\ s=4;\ \Delta = 400-16$

$\Delta > 0;\ r > 0 \Rightarrow \boxed{M_3 \rightarrow \text{лок. мин.}}$ ✓

(10) $z(x,y) = xy(6-x-y) = 6xy - x^2y - xy^2$

$z_x = 6y - 2xy - y^2$ $\qquad 6y - 2xy - y^2 = 0$

$z_y = 6x - x^2 - 2xy$ $\qquad 6x - x^2 - 2xy = 0$

$6y - \cancel{2xy} - y^2 = 6x - x^2 - \cancel{2xy}$

$6y - 6x = y^2 - x^2$

$6(y-x) = (y-x)(y+x)$

$6(y-x) - (y-x)(y+x) = 0$

$(y-x)(6-y-x) = 0$

1° $y - x = 0 \Rightarrow \boxed{y = x}$

$6x - x^2 - 2x^2 = 0$

$-3x^2 + 6x = 0$

$-3x(x-2) = 0$

$x = 0 \ \vee \ x = 2$

$y = 0 \qquad y = 2$

$\boxed{M_1(0,0)}$ $\quad \boxed{M_2(2,2)}$

2° $6 - y - x = 0 \Rightarrow \boxed{y = 6-x}$

$6x - x^2 - 2x(6-x) = 0$

$6x - x^2 - 12x + 2x^2 = 0$

$x^2 - 6x = 0$

$x(x-6) = 0$

$x = 0 \ \vee \ x = 6$

$y = 6 \qquad y = 0$

$\boxed{M_3(0,6)}$ $\quad \boxed{M_4(6,0)}$

$$\boxed{\begin{array}{l} z_{xx} = -2y \quad r \\ z_{yy} = -2x \quad t \\ z_{xy} = 6-2x-2y \quad s \end{array}}$$

$M_1(0,0)$

$r=0;\ t=0;\ s=6;\ \Delta=-36$

$\Delta < 0 \Rightarrow \boxed{M_1 \rightarrow \text{није екстрем}}$

$M_2(2,2)$

$r=-4;\ t=-4;\ s=-2;\ \Delta = 12 \Rightarrow \boxed{M_2 \rightarrow \text{лок. макс}}$

(11)

$M_3(0,6)$

$r=-12;\ t=0;\ s=-6;\ \Delta=-36$

$\Delta<0 \Rightarrow \boxed{M_3 \to \text{НИЈЕ ЕКСТРЕМ}}$

$M_4(6,0)$

$r=0;\ t=-12;\ s=-6;\ \Delta=-36 \Rightarrow \boxed{M_4 \to \text{НИЈЕ ЕКСТРЕМ}}$

11. $z(x,y)=x^4-2x^2y-6x^2+4y^2$

$z_x=4x^3-4xy-12x$ $\qquad 4x^3-4xy-12x=0$

$z_y=-2x^2+8y$ $\qquad -2x^2+8y=0$

$x^3-xy-3x=0$

$x(x^2-y-3)=0$

$1^\circ\ x=0,\ y=0 \quad \boxed{M_1(0,0)}$

$2^\circ\ x^2-y-3=0 \Rightarrow y=x^2-3$

$-2x^2+8(x^2-3)=0$

$-2x^2+8x^2-24=0$

$6x^2-24=0$

$x^2-4=0$

$(x-2)(x+2)=0$

$x=2 \lor x=-2$

$y=1 \qquad y=1$

$\boxed{M_2(2,1)} \quad \boxed{M_3(-2,1)}$

$M_1(0,0)$

$r=-12;\ t=8;\ s=0;\ \Delta=-12\cdot 8$

$\Delta<0 \Rightarrow \boxed{M_1 \to \text{НИЈЕ ЕКСТРЕМ}}$

$$\boxed{\begin{array}{ll} z_{xx}=12x^2-4y-12 & r \\ z_{yy}=8 & t \\ z_{xy}=-4x & s \end{array}}$$

$M_2(2,1)$

$r=32;\ t=8;\ s=-8;\ \Delta=4\cdot 64-64=3\cdot 64$

$\Delta>0;\ r>0 \Rightarrow \boxed{M_1 \to \text{лок. мин}}$

$M_3(-2,1)$

$r=32;\ t=8;\ s=+8;\ \Delta=3\cdot 64$

$\Delta>0;\ r>0 \Rightarrow \boxed{M_2 \to \text{лок. мин.}}$

(12.) $z(x,y) = x^4 + y^4 - 4xy$

$z_x = 4x^3 - 4y$ $\qquad 4x^3 - 4y = 0$

$z_y = 4y^3 - 4x$ $\qquad 4y^3 - 4x = 0$

$y = x^3$

$4x^6 - 4x = 0$

$x^6 - x = 0$

$x(x^5 - 1) = 0$

$x = 0 \quad \vee \quad x^5 = 1$

$y = 0 \qquad x = 1$

$\boxed{M_1(0,0)} \qquad y = 1$

$\boxed{M_2(1,1)}$

$\boxed{\begin{array}{l|l} z_{xx} = 12x^2 & r \\ z_{yy} = 12y^2 & t \\ z_{xy} = -4 & s \end{array}}$

$M_1(0,0)$

$r = 0;\ t = 0;\ s = -4;\ \Delta = -16$

$\Delta < 0 \Rightarrow$ $\boxed{M_1 \to \text{НИЈЕ ЕКСТРЕМ}}$

$M_2(1,1)$

$r = 12;\ t = 12;\ s = -4;\ \Delta = 144 - 16$

$\Delta > 0;\ r > 0 \Rightarrow$ $\boxed{M_2 \to \text{лок. мин.}}$

(13.) $z(x,y) = (y - 2x + 5)e^{x^2 - y}$

$z_x = -2 \cdot e^{x^2-y} + (y - 2x + 5) \cdot e^{x^2-y} \cdot 2x$

$z_y = e^{x^2-y} + (y - 2x + 5) \cdot e^{x^2-y}(-1)$

$-e^{x^2-y} + x(y - 2x + 5) \cdot e^{x^2-y} = 0$

$e^{x^2-y} - (y - 2x + 5) \cdot e^{x^2-y} = 0$ $\quad +$

$e^{x^2-y}\left(x(y - 2x + 5) - (y - 2x + 5)\right) = 0$

$(y - 2x + 5)(x - 1) = 0$

1° $x = 1$

$e^{x^2-y}(1 - y + 2 - 5) = 0$

$-y - 2 = 0$

$y = -2 \Rightarrow \boxed{M_1(1,-2)}$

2° $y - 2x + 5 = 0$

$y = 2x - 5$

$e^{x^2-y}(1 - y + 2x - 5) = 0$

$1 - \cancel{2x} + 5 + \cancel{2x} - 5 = 0$

$0 = 0$

(14.

$$z_{xx} = e^{x^2-y}\cdot 2x(xy-2x^2+5x-1) + e^{x^2-y}(y-4x+5) =$$

$$= e^{x^2-y}\left(2x(xy-2x^2+5x-1)+y-4x+5\right) =$$

$$= e^{x^2-y}\left(2x^2y-4x^3+10x^2-2x+y-4x+5\right) =$$

$$= \boxed{e^{x^2-y}\left(-4x^3+10x^2-6x+y+2x^2y+5\right)}$$

$$z_{yy} = e^{x^2-y}(-1)(-y+2x-4) + e^{x^2-y}(-1) =$$

$$\boxed{= -e^{x^2-y}(-y+2x-4) =}$$

$$z_{xy} = e^{x^2-y}(-1)(xy-2x^2+5x-1) + e^{x^2-y}\cdot x =$$

$$= e^{x^2-y}(-xy+2x^2-5x+1+x)$$

$$\boxed{= e^{x^2-y}(2x^2-4x-xy+1)}$$

$M(1,-2)$

$$r = e^3(\cancel{-4}+\cancel{10}-\cancel{6}-2-4+5) = e^3\cdot(-1) = -e^3$$

$$t = -e^3(\cancel{2}+\cancel{2}-\cancel{4}) = 0$$

$$s = e^3(\cancel{2}-\cancel{4}+\cancel{2}+1) = e^3$$

$$\Delta = -e^6 \Rightarrow \Delta < 0 \Rightarrow \boxed{M \to \text{није екстрем}}$$

(14.) $z(x,y) = (x^2+y^2)e^{x-y}$

$$z_x = 2x\cdot e^{x-y} + (x^2+y^2)e^{x-y} = \boxed{e^{x-y}(x^2+y^2+2x)}$$

$$z_y = 2y\cdot e^{x-y} + (x^2+y^2)e^{x-y}(-1) = \boxed{e^{x-y}(-x^2-y^2+2y)}$$

$$e^{x-y}(x^2+y^2+2x) = 0$$

$$e^{x-y}(-x^2-y^2+2y) = 0$$

$e^{x-y} \neq 0 \quad \forall x,y \in R$

$$x^2+y^2+2x = 0$$

$$-x^2-y^2+2y = 0$$

$$2x+2y = 0$$

$$\boxed{x = -y}$$

$$e^{2x}(x^2+x^2+2x) = 0$$

$$2x^2+2x = 0$$

$$2x(x+1) = 0$$

$x = 0$, $y = 0$ $\Rightarrow \boxed{M_1(0,0)}$ $\vee$ $x = -1$, $y = 1$ $\Rightarrow \boxed{M_2(-1,1)}$

$$z_{xx} = e^{x-y}(x^2+y^2+2x) + e^{x-y}(2x+2) =$$
$$= e^{x-y}(x^2+y^2+2x+2x+2)$$
$$= \boxed{e^{x-y}(x^2+y^2+4x+2)}$$

$$z_{yy} = -e^{x-y}(-x^2-y^2+2y) + e^{x-y}(-2y+2)$$
$$= -e^{x-y}(-x^2-y^2+\cancel{2y}-\cancel{2y}+2)$$
$$\boxed{= -e^{x-y}(-x^2-y^2+2)}$$

$$z_{xy} = -e^{x-y}(x^2+y^2+2x) + e^{x-y}(2y)$$
$$= \boxed{e^{x-y}(-x^2-y^2-2x+2y)}$$

$M_1(0,0)$

$\downarrow$

$r = 2$; $t = -2$; $s = 0$; $\Delta = -4$

$\Delta < 0 \Longrightarrow$ $\boxed{M_1 \text{ – ниже екстрем}}$

$M_2(-1,1)$

$\downarrow$

$r = e^{-2}(\cancel{1}+\cancel{1}-\cancel{4}+2) = e^{-2}\cdot 0 = 0$

$t = -e^{-2}(-\cancel{1}-\cancel{1}+2) = 0$

$s = e^{-2}(-1-1+2+2) = 2e^{-2}$

$\Delta = -4e^{-4} \Rightarrow \Delta < 0 \Rightarrow$ $\boxed{M_2 \text{ – ниже екстрем}}$ ✓

(15) $z(x,y) = (x+1)^2 \cdot y \cdot e^{y-x}$

$$z_x = 2y(x+1)\cdot e^{y-x} + y(x+1)^2 \cdot e^{y-x}(-1) =$$
$$= e^{y-x}\left(2y(x+1) - y(x+1)^2\right)$$
$$= e^{y-x}\left(y(x+1)(2-x-1)\right)$$
$$= \boxed{e^{y-x}\left(y\cdot(1-x^2)\right)}$$

$$z_y = (x+1)^2 \cdot e^{y-x} + y(x+1)^2 \cdot e^{y-x}$$
$$= e^{y-x}\left((x+1)^2 + y(x+1)^2\right)$$
$$= \boxed{e^{y-x}\cdot (x+1)^2 (y+1)}$$

$e^{y-x}(y\cdot(1-x^2))=0$

$e^{y-x}(x+1)^2(y+1)=0$

$y(1-x^2)=0$

$y(1-x)(1+x)=0$

$1^\circ\ y=0$
$(x+1)^2=0$
$x+1=0$
$x=-1$
$\boxed{M_1(-1,0)}$

$2^\circ\ x=1$
$(y+1)\cdot 4=0$
$y=-1$
$\boxed{M_2(1,-1)}$

$3^\circ\ x=-1$
$y\in R$
$\boxed{M_3(-1,y);\ y\in R}$

$z_{xx}=-e^{y-x}(y\cdot(1-x^2))+e^{y-x}\cdot y\cdot(-2x)$

$=e^{y-x}(y(x^2-1)-2xy)$

$=\boxed{e^{y-x}(x^2y-2xy-y)}$

$z_{yy}=e^{y-x}(x+1)^2(y+1)+e^{y-x}(x+1)^2$

$=\boxed{e^{y-x}((x+1)^2(y+2))}$

$z_{xy}=e^{y-x}(y(1-x^2))+e^{y-x}(1-x^2)$

$=\boxed{e^{y-x}((1-x^2)(y+1))}$

$\underline{M_2(1,-1)}$

$r=e^{-2}(-1+2+1)=2e^{-2}$

$t=e^{-2}(4\cdot 1)=4e^{-2}$

$s=e^{-2}\cdot 0=0$

$\Delta=8e^{-4}$

$\Delta>0,\ r>0\Rightarrow$ $\boxed{M_2 \to \text{лок. мин.}}$

$\underline{M_1(-1,0)}$

$r=0;\ t=0;\ s=0;\ \Delta=0$

$\Delta z(-1,0)=z(-1+\Delta x,0+\Delta y)-z(-1,0)$

$\Delta z(-1,0)=\Delta x^2\cdot\Delta y\cdot e^{\Delta y-\Delta x+1}$

прираштај

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16. $z(x,y) = (x^2-2y^2)\,e^{x-y}$

$z_x = 2x\cdot e^{x-y} + (x^2-2y^2)\cdot e^{x-y}$

$= \boxed{e^{x-y}(x^2-2y^2+2x)}$

$z_y = -4y\cdot e^{x-y} + (x^2-2y^2)\cdot e^{x-y}(-1)$

$= \boxed{-e^{x-y}(x^2-2y^2+4y)}$

$$\left.\begin{aligned} e^{x-y}(x^2-2y^2+2x) &= 0 \\ -e^{x-y}(x^2-2y^2+4y) &= 0 \end{aligned}\right] +$$

$e^{x-y}(\cancel{x^2}-\cancel{2y^2}+2x-\cancel{x^2}+\cancel{2y^2}-4y) = 0$

$e^{x-y}(2x-4y) = 0$

$x-2y=0$

$\boxed{x=2y}$

$e^{x-y}(4y^2-2y^2+4y)=0$

$2y^2+4y=0$

$2y(y+2)=0$

$y=0 \quad \vee \quad y=-2$

$x=0 \qquad x=-4$

$\boxed{M_1(0,0)} \qquad \boxed{M_2(-4,-2)}$

$z_{xx} = e^{x-y}(x^2-2y^2+2x) + e^{x-y}(2x+2)$

$= e^{x-y}(x^2-2y^2+2x+2x+2)$

$= \boxed{e^{x-y}(x^2-2y^2+4x+2)}$

$z_{yy} = -e^{x-y}(-1)(x^2-2y^2+4y) - e^{x-y}(-4y+4)$

$= e^{x-y}(x^2-2y^2+4y+4y-4)$

$= \boxed{e^{x-y}(x^2-2y^2+8y-4)}$

$z_{xy} = e^{x-y}(-1)(x^2-2y^2+2x) + e^{x-y}(-4y)$

$= \boxed{e^{x-y}(-x^2+2y^2-2x-4y)}$

$\underline{M_1(0,0)}$

$r=2;\ t=-4;\ s=0;\ \Delta=-8$

$\Delta<0 \Rightarrow$ $\boxed{M_1 \rightarrow \text{НИЈЕ ЕКСТРЕМ}}$

$\underline{M_2(-4,-2)}$

$r=(\cancel{16}-8-\cancel{16}+2)e^{-2} = -6e^{-2}$

$t=(16-8-16-4)e^{-2} = -12e^{-2}$

$s=(-\cancel{16}+\cancel{8}+8+8)e^{-2} = 8e^{-2}$

$\Delta = 72e^{-4} - 64e^{-4} = 8e^{-4}$

$\Delta>0;\ r<0 \Rightarrow$ $\boxed{M_2 \rightarrow \text{ЛОК МАКС}}$

11. $z(x,y) = (x^2+y)\sqrt{e^y}$

$z_x = 2x\cdot\sqrt{e^y}$

$z_y = \sqrt{e^y} + (x^2+y)\cdot\frac{1}{2\sqrt{e^y}}\cdot e^y$

$= \sqrt{e^y} + (x^2+y)\cdot\frac{\sqrt{e^y}}{2}$

$= \sqrt{e^y}\left(1+\frac{x^2+y}{2}\right) = \frac{\sqrt{e^y}(x^2+y+2)}{2}$

$2x\cdot\sqrt{e^y} = 0$

$\sqrt{e^y}(x^2+y+2) = 0$

1° $x = 0$

$y+2 = 0$

$y = -2$

$M_1(0,-2)$

2° $x^2+y+2 = 0$

$x^2 = -y-2$

$x = \pm\sqrt{-y-2}$

$2\sqrt{(-y-2)e^y} = 0 \quad /^2$

$e^y(-y-2) = 0$

$y = -2$

$x = 0$

$z_{xx} = 2\sqrt{e^y}$

$z_{yy} = \frac{1}{2}\left(\frac{\sqrt{e^y}}{2}(x^2+y+2) + \sqrt{e^y}\right)$

$= \frac{1}{2}\left(\sqrt{e^y}\left(\frac{x^2+y+2}{2}+1\right)\right)$

$= \frac{\sqrt{e^y}(x^2+y+4)}{4}$

$z_{xy} = \cancel{2}x\cdot\frac{\sqrt{e^y}}{\cancel{2}} = x\cdot\sqrt{e^y}$

$M(0,-2)$

$\downarrow$

$r = \frac{2}{e}$

$t = \frac{1}{e}(-2+4) = \frac{2}{e}$

$s = 0$

$\Delta = \frac{4}{e^2}$

$\Delta > 0;\ r > 0 \Rightarrow \boxed{M - \text{лок. мин.}}$

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18. $z(x,y) = (x+2y+y^2)e^{2x}$

$z_x = e^{2x} + (x+2y+y^2)e^{2x}\cdot 2$

$= \boxed{e^{2x}(2x+4y+2y^2+1)}$

$z_y = \boxed{e^{2x}(2y+2)}$

$e^{2x}(2y+2) = 0$

$e^{2x}(2x+4y+2y^2+1) = 0$

$2y+2 = 0$

$y = -1$

$2x-4+2+1 = 0$

$2x-1 = 0$

$x = \frac{1}{2}$

$\boxed{M = \left(\frac{1}{2}, -1\right)}$

$z_{xx} = 2e^{2x}(2x+4y+2y^2+1) + e^{2x}(2)$

$= \boxed{2e^{2x}(2x+4y+2y^2+2)}$

$z_{yy} = \boxed{e^{2x}\,2}$

$z_{xy} = \boxed{e^{2x}(4+4y)}$

$M\left(\frac{1}{2}, -1\right)$

$r = (1-\cancel{4}+\cancel{2}+\cancel{2})\,2\cdot e = 2e$

$t = 2e$

$s = e(4-4) = 0$

$\Delta = 4e^2$

$\Delta > 0;\ r > 0 \Rightarrow$ $\boxed{M \to \text{лок. мин.}}$

19. $z(x,y) = x+y+\frac{1}{x}+\frac{4}{y}$

$z_x = 1-\frac{1}{x^2}$

$z_y = 1-\frac{4}{y^2}$

$1-\frac{1}{x^2} = 0$

$1-\frac{4}{y^2} = 0$

$\frac{1}{x^2} = 1$

$x^2 = 1$

$x = \pm 1$

$\frac{4}{y^2} = 1$

$y^2 = 4$

$y = \pm 2$

$\boxed{M_1(1,2)}$

$\boxed{M_2(1,-2)}$

$\boxed{M_3(-1,2)}$

$\boxed{M_4(-1,-2)}$

$z_{xx} = \frac{2}{x^3}$

$z_{yy} = \frac{8}{y^3}$

$z_{xy} = 0$

$M_1(1,2)$

$r = 2;\ t = 1;\ s = 0;\ \Delta = 2$

$\Delta > 0,\ r > 0 \Rightarrow$ $\boxed{M_1 \to \text{лок. мин.}}$

$M_2(1,-2)$

$r = 2;\ t = -1;\ s = 0,\ \Delta = -2$

$\Delta < 0 \Rightarrow$ $\boxed{M_2 \to \text{није екстрем}}$

$M_3(-1,2)$

$r = -2,\ t = 1;\ s = 0;\ \Delta = -2$

$\Delta < 0 \Rightarrow$ $\boxed{M_3 \to \text{није екстрем}}$

$M_4(-1,-2)$

$r = -2,\ t = -1;\ s = 0;\ \Delta = 2$

$\Delta > 0,\ r < 0 \Rightarrow$ $\boxed{M_4 \to \text{лок. макс}}$

10. $z(x,y) = x^2+y^2+\frac{4}{x}+\frac{4}{y}+2xy$

$z_x = 2x - \frac{4}{x^2} + 2y$ $\qquad 2x - \frac{4}{x^2} + 2y = 0$

$z_y = 2y - \frac{4}{y^2} + 2x$ $\qquad 2y - \frac{4}{y^2} + 2x = 0$

$$\cancel{2x} - \frac{4}{x^2} + \cancel{2y} = \cancel{2y} - \frac{4}{y^2} + \cancel{2x}$$

$$\frac{4}{y^2} - \frac{4}{x^2} = 0$$

$$4\left(\frac{1}{y^2} - \frac{1}{x^2}\right) = 0$$

$$\frac{1}{x^2} = \frac{1}{y^2}$$

$$x^2 = y^2$$

$$\boxed{x = \pm y}$$

1° $x = y$

$$2x - \frac{4}{x^2} + 2x = 0$$

$$4x - \frac{4}{x^2} = 0$$

$$4x\left(1 - \frac{1}{x^3}\right) = 0$$

$x = 0 \ \lor \ \frac{1}{x^3} = 1$

$y = 0$ $\qquad x^3 = 1$

$\boxed{M_1(0,0)}$ $\qquad x = 1$

$y = 1$

$\boxed{M_2(1,1)}$

2° $x = -y$

$$\cancel{2x} - \frac{4}{x^2} - \cancel{2x} = 0$$

$$-\frac{4}{x^2} = 0 \quad //$$

$$\boxed{\begin{aligned} z_{xx} &= 2 + \frac{8}{x^3} \\ z_{yy} &= 2 + \frac{8}{y^3} \\ z_{xy} &= 2 \end{aligned}}$$

$\underline{M_1(0,0)}$

$\downarrow$

$r = 2;\ t = 2;\ s = 2;\ \Delta = 0$ $\qquad$ прираштај

$\Delta z(0,0) = z(\Delta x, \Delta y) - z(0,0)$

$\Delta z(0,0) = \Delta x^2 + \Delta y^2 + \frac{4}{\Delta x} + \frac{4}{\Delta y} + 2\Delta x \Delta y$

$\underline{M_2(1,1)}$

$\downarrow$

$r = 10;\ t = 10;\ s = 2;\ \Delta = 96$

$\Delta > 0;\ r > 0 \Rightarrow \boxed{M_2 \to \text{лок. мин.}}$

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(21) $z(x,y)=\dfrac{x+y+1}{\sqrt{x^2+y^2+1}}$

$$z_x=\frac{\sqrt{x^2+y^2+1}-(x+y+1)\cdot\dfrac{1}{\cancel{2}\sqrt{x^2+y^2+1}}\cdot\cancel{2}x}{x^2+y^2+1}$$

$$z_x=\frac{\sqrt{x^2+y^2+1}-\dfrac{x(x+y+1)}{\sqrt{x^2+y^2+1}}}{x^2+y^2+1}=\frac{\dfrac{x^2+y^2+1-x(x+y+1)}{\sqrt{x^2+y^2+1}}}{x^2+y^2+1}$$

$$=\frac{\cancel{x^2}+y^2+1-\cancel{x^2}-xy-x}{(x^2+y^2+1)^{3/2}}=\boxed{\frac{y^2-xy-x+1}{(x^2+y^2+1)^{3/2}}}$$

$$z_y=\frac{\sqrt{x^2+y^2+1}-(x+y+1)\cdot\dfrac{\cancel{2}y}{\cancel{2}\sqrt{x^2+y^2+1}}}{x^2+y^2+1}=\frac{\dfrac{x^2+y^2+1-y(x+y+1)}{\sqrt{x^2+y^2+1}}}{x^2+y^2+1}=$$

$$=\frac{x^2+\cancel{y^2}+1-xy-\cancel{y^2}-y}{(x^2+y^2+1)^{3/2}}=\boxed{\frac{x^2-xy-y+1}{(x^2+y^2+1)^{3/2}}}$$

$$\frac{y^2-xy-x+1}{(x^2+y^2+1)^{3/2}}=0 \qquad\qquad x^2+y^2+1\neq 0$$

$$\frac{x^2-xy-y+1}{(x^2+y^2+1)^{3/2}}=0$$

$$\frac{y^2-xy-x+1}{(x^2+y^2+1)^{3/2}}=\frac{x^2-xy-y+1}{(x^2+y^2+1)^{3/2}}$$

$$(x^2+y^2+1)^{3/2}(y^2-xy-x+1)=(x^2+y^2+1)^{3/2}(x^2-xy-y+1)$$

$$(x^2+y^2+1)^{3/2}(y^2\cancel{-xy}-x\cancel{+1}-x^2\cancel{+xy}+y\cancel{-1})=0$$

$$(x^2+y^2+1)^{3/2}(y^2-x^2-x+y)=0$$

$$y^2-x^2-x+y=0$$

$$(y-x)(y+x)+(y-x)=0$$

$$(y-x)(y+x+1)=0$$

1° $y-x=0 \Rightarrow y=x$ $\qquad\qquad$ 2° $y+x+1=0$

$$\frac{\cancel{x^2}-\cancel{x^2}-x+1}{(x^2+x^2+1)^{3/2}}=0 \qquad\qquad y=-x-1$$

$$\frac{1-x}{(2x^2+1)^{3/2}}=0 \qquad\qquad \frac{x^2-x(-x-1)-(-x-1)+1}{(x^2+(-x-1)^2+1)^{3/2}}=0$$

$$x=1 \qquad\qquad \frac{x^2+x^2+x+x+1+1}{(x^2+x^2+2x+1+1)^{3/2}}=\frac{2(x^2+x+1)}{(2x^2+2x+2)^{3/2}}=0$$

$$y=1 \qquad\qquad x^2+x+1=0$$

$$\boxed{M_1(1,1)} \qquad\qquad x_{1/2}=\frac{-1\pm\sqrt{1-4}}{2}\quad\perp$$

$$z_{xx} = \frac{(-y-1)(x^2+y^2+1)^{3/2} - (y^2-xy-x+1)\cdot\frac{3}{2}(x^2+y^2+1)^{\frac{1}{2}}\cdot 2x}{(x^2+y^2+1)^3} =$$

$$= \frac{\cancel{(x^2+y^2+1)^{1/2}}\left((-y-1)(x^2+y^2+1) - 3x(y^2-xy-x+1)\right)}{(x^2+y^2+1)^{\cancel{3}\,\frac{5}{2}}} =$$

$$= \frac{-(y+1)(x^2+y^2+1) - 3x(y^2-xy-x+1)}{(x^2+y^2+1)^{5/2}} = \frac{-2(3) - 3(\cancel{1}-\cancel{1}-\cancel{1}+\cancel{1})}{3^{5/2}} = \frac{-2\cdot 3}{3^{5/2}} = -\frac{2}{3\sqrt{3}} = -\frac{2\sqrt{3}}{9}$$

$$z_{yy} = \frac{(-x-1)(x^2+y^2+1)^{3/2} - (x^2-xy-y+1)\frac{3}{2}(x^2+y^2+1)^{\frac{1}{2}}\cdot 2y}{(x^2+y^2+1)^3} =$$

$$= \frac{(x^2+y^2+1)^{1/2}\left(-(x+1)(x^2+y^2+1) - 3y(x^2-xy-y+1)\right)}{(x^2+y^2+1)^3} =$$

$$= \frac{-(x+1)(x^2+y^2+1) - 3y(x^2-xy-y+1)}{(x^2+y^2+1)^{5/2}} = -\frac{2\sqrt{3}}{9}$$

$$z_{xy} = \frac{(2y-x)(x^2+y^2+1)^{3/2} - (y^2-xy-x+1)\cdot\frac{3}{2}(x^2+y^2+1)^{1/2}\cdot 2y}{(x^2+y^2+1)^3} =$$

$$= \frac{(x^2+y^2+1)^{\frac{1}{2}}\left((2y-x)(x^2+y^2+1) - 3y(y^2-xy-x+1)\right)}{(x^2+y^2+1)^3} =$$

$$= \frac{(2y-x)(x^2+y^2+1) - 3y(y^2-xy-x+1)}{(x^2+y^2+1)^{5/2}} = \frac{3-0}{3\sqrt{3}} = \frac{3\sqrt{3}}{9} = \frac{\sqrt{3}}{3}$$

$\underline{M(1,1)}$

$r = -\frac{2\sqrt{3}}{9}$; $t = -\frac{2\sqrt{3}}{9}$; $s = \frac{\sqrt{3}}{3}$

$$\Delta = \frac{12}{81} - \frac{3}{9} = \frac{12}{81} - \frac{27}{81} = -\frac{15}{81}$$

$\Delta < 0 \Rightarrow$ $\boxed{\text{M – НИЈЕ ЕКСТРЕМ}}$

(22.) $z(x,y) = \frac{x+y-1}{\sqrt{x^2+y^2+1}}$ $\quad M(-1,-1)$

(23.) $z(x,y) = xy\ln(x^2+y^2)$

$$z_x = y\cdot\ln(x^2+y^2) + xy\cdot\frac{1}{x^2+y^2}\cdot 2x$$
$$= y\cdot\ln(x^2+y^2) + \frac{2x^2y}{x^2+y^2}$$

$$z_y = x\cdot\ln(x^2+y^2) + xy\cdot\frac{1}{x^2+y^2}\cdot 2y$$
$$= x\cdot\ln(x^2+y^2) + \frac{2xy^2}{x^2+y^2}$$

$$y\cdot \ln(x^2+y^2)+\frac{2x^2y}{x^2+y^2}=0$$

$$x\ln(x^2+y^2)+\frac{2xy^2}{x^2+y^2}=0$$

$$\sqrt{\frac{1}{2e}}\ln\left(\frac{1}{2e}+\frac{1}{2e}\right)+\frac{2\cdot\frac{1}{2e}\sqrt{\frac{1}{2e}}}{\frac{1}{2e}+\frac{1}{2e}}=$$

$$=\sqrt{\frac{1}{2e}}\ln\frac{1}{e}+\frac{\cancel{\frac{1}{e}}\sqrt{\frac{1}{2e}}}{\cancel{\frac{1}{e}}}$$

$$=\sqrt{\frac{1}{2e}}(-1)+\sqrt{\frac{1}{2e}}=0$$

$$y\ln(x^2+y^2)=-\frac{2x^2y}{x^2+y^2}$$

$$x\ln(x^2+y^2)=-\frac{2xy^2}{x^2+y^2}$$

$$\ln(x^2+y^2)=-\frac{2x^2y}{y(x^2+y^2)}$$

$$\ln(x^2+y^2)=-\frac{2xy^2}{x(x^2+y^2)}$$

$$\frac{2x^2y}{y(x^2+y^2)}=\frac{2xy^2}{x(x^2+y^2)}$$

$$2xy^3(x^2+y^2)=2x^3y(x^2+y^2)$$

$$(x^2+y^2)(2xy^3-2x^3y)=0$$

$$(x^2+y^2)(2xy(y^2-x^2))=0$$

$$2xy(x^2+y^2)(y-x)(y+x)=0$$

1° $x=0$, $y\in R$

2° $x^2+y^2=0$, $x=0$, $y=0$, $\cancel{\boxed{M_1(0,0)}}$

Нује у домену

3° $y-x=0$

$y=x$

$$x\ln(2x^2)+\frac{\cancel{2}x^3}{\cancel{2x^2}}=0$$

$$x(\ln 2x^2+1)=0$$

$x=0 \quad \vee \quad \ln 2x^2=-1$

$y=0$ нсо $\quad e^{-1}=2x^2$

$$x^2=\frac{1}{2e}$$

$x=\sqrt{\frac{1}{2e}} \quad \wedge \quad x=-\sqrt{\frac{1}{2e}}$

$y=\sqrt{\frac{1}{2e}} \quad\quad y=-\sqrt{\frac{1}{2e}}$

$$\boxed{M_2\left(\sqrt{\frac{1}{2e}},\sqrt{\frac{1}{2e}}\right)} \quad \boxed{M_3\left(-\sqrt{\frac{1}{2e}},-\sqrt{\frac{1}{2e}}\right)}$$

4° $y=-x$

$$x\ln 2x^2+\frac{2x^3}{2x^2}=0$$

нсо

$$Z_{xx} = y \cdot \frac{2x}{x^2+y^2} + \frac{4xy(x^2+y^2) - 2x^2y \cdot 2x}{(x^2+y^2)^2} =$$

$$= \frac{2xy}{x^2+y^2} + \frac{\cancel{4x^3y} + 4xy^3 - \cancel{4x^3y}}{(x^2+y^2)^2} = \frac{2xy(x^2+y^2) + 4xy^3}{(x^2+y^2)^2} =$$

$$= \frac{2xy(x^2+y^2+2y^2)}{(x^2+y^2)^2} = \boxed{\frac{2xy(x^2+3y^2)}{(x^2+y^2)^2}}$$

$$Z_{yy} = x \cdot \frac{2y}{x^2+y^2} + \frac{4xy(x^2+y^2) - 2xy^2 \cdot 2y}{(x^2+y^2)^2} =$$

$$= \frac{2xy(x^2+y^2) + 4xy(x^2+y^2) - 4xy^3}{(x^2+y^2)^2} = \frac{6xy(x^2+y^2) - 4xy^3}{(x^2+y^2)^2} =$$

$$= \frac{2xy(3x^2+3y^2-2y^2)}{(x^2+y^2)^2} = \boxed{\frac{2xy(3x^2+y^2)}{(x^2+y^2)^2}}$$

$$Z_{xy} = \ln(x^2+y^2) + y \cdot \frac{2y}{x^2+y^2} + \frac{2x^2(x^2+y^2) - 2x^2y \cdot 2y}{(x^2+y^2)^2} =$$

$$= \ln(x^2+y^2) + \frac{2y^2(x^2+y^2) + 2x^2(x^2+y^2) - 4x^2y^2}{(x^2+y^2)^2} =$$

$$= \boxed{\ln(x^2+y^2) + \frac{2(x^2+y^2)^2 - 4x^2y^2}{(x^2+y^2)^2}}$$

$M_1(0,0)$

$r = 0$; $t = 0$; $s = \ln 0$?

$M_2\left(\sqrt{\frac{1}{2e}}, \sqrt{\frac{1}{2e}}\right)$

$$r = \frac{\frac{1}{e}\left(\frac{1}{2e} + \frac{3}{2e}\right)}{\frac{1}{e^2}} = \frac{\frac{1}{e} \cdot \frac{2}{e}}{\frac{1}{e^2}} = 2$$

$t = 2$

$$s = \ln\left(\frac{1}{e}\right) + \frac{2 \cdot \frac{1}{e^2} - 4 \cdot \frac{1}{4e^2}}{\frac{1}{e^2}} = -1 + 1 = 0$$

$\Delta = 4$

$\Delta > 0$; $r > 0$ $\Rightarrow$ $\boxed{M_2 \to \text{лок. мин.}}$

$\boxed{M_3 \to \text{лок. мин.}}$

24. $z(x,y) = 2x + 2y - \ln(x^2+y^2+1)^3$

$$z_x = 2 - \frac{1}{(x^2+y^2+1)^3} \cdot 3(x^2+y^2+1)^2 \cdot 2x = 2 - \frac{6x}{x^2+y^2+1}$$

$$z_y = 2 - \frac{1}{(x^2+y^2+1)^3} \cdot 3(x^2+y^2+1)^2 \cdot 2y = 2 - \frac{6y}{x^2+y^2+1}$$

$$2 - \frac{6x}{x^2+y^2+1} = 0$$

$$2 - \frac{6y}{x^2+y^2+1} = 0$$

$$\frac{6x}{x^2+y^2+1} = \frac{6y}{x^2+y^2+1}$$

$$6y(x^2+y^2+1) - 6x(x^2+y^2+1) = 0$$

$$6(x^2+y^2+1)(y-x) = 0$$

$$y - x = 0$$

$$y = x$$

$$2 - \frac{6x}{2x^2+1} = 0$$

$$\frac{6x}{2x^2+1} = 2$$

$$3x = 2x^2+1$$

$$2x^2 - 3x + 1 = 0$$

$$x_{1/2} = \frac{3 \pm \sqrt{9-8}}{4} = \frac{3 \pm 1}{4} = \begin{cases} 1 \\ +\frac{1}{2} \end{cases}$$

$\boxed{M_1(1,1)}$ $\boxed{M_2(+\frac{1}{2}, +\frac{1}{2})}$

$$z_{xx} = \frac{-6(x^2+y^2+1) + 6x(2x)}{(x^2+y^2+1)^2} = \frac{6(x^2-y^2-1)}{(x^2+y^2+1)^2}$$

$$z_{yy} = \frac{-6(x^2+y^2+1) + 6y \cdot 2y}{(x^2+y^2+1)^2} = \frac{6(-x^2+y^2-1)}{(x^2+y^2+1)^2}$$

$$z_{xy} = \frac{6x}{(x^2+y^2+1)^2} \cdot 2y = \frac{12xy}{(x^2+y^2+1)^2}$$

$\frac{\frac{3}{9}}{4}$ $\frac{12}{9} = \frac{4}{3}$

$M_1(1,1)$

$r = -\frac{2}{3};\ t = -\frac{2}{3};\ s = \frac{4}{3};\ \Delta = -\frac{4}{3}$

$\Delta < 0 \Rightarrow$ $\boxed{M_1 \to \text{НИЈЕ ЕКСТРЕМ}}$

$M_2(+\frac{1}{2}, +\frac{1}{2})$

$r = -\frac{8}{3};\ t = -\frac{8}{3};\ s = \frac{4}{3};\ \Delta = \frac{64-16}{9}$

$\Delta > 0;\ r < 0 \Rightarrow$ $\boxed{M_2 \to \text{лок. макс.}}$

25) $z(x,y) = x + \frac{8}{y} + \frac{y}{x}$

$z_x = 1 - \frac{y}{x^2}$

$z_y = -\frac{8}{y^2} + \frac{1}{x}$

$1 - \frac{y}{x^2} = 0$

$-\frac{8}{y^2} + \frac{1}{x} = 0$

$\frac{y}{x^2} = 1$

$\boxed{y = x^2}$

$-\frac{8}{x^4} + \frac{1}{x} = 0$

$\frac{1}{x}\left(1 - \frac{8}{x^3}\right) = 0$

$1 - \frac{8}{x^3} = 0$

$\frac{8}{x^3} = 1$

$x^3 = 8$

$x = 2$ $\quad M_1(2,4)$

$y = 4$

$$\boxed{\begin{aligned} z_{xx} &= \frac{2y}{x^3} \\ z_{yy} &= \frac{16}{y^3} \\ z_{xy} &= -\frac{1}{x^2} \end{aligned}}$$

$M(2,4)$

$r = 1$; $t = \frac{1}{4}$; $s = -\frac{1}{4}$; $\Delta = \frac{3}{16}$

$\Delta > 0$; $r > 0 \Rightarrow$ $\boxed{M \text{ - лок. мин.}}$

26) $z(x,y) = x^2 + xy + y^2 + \frac{3}{x} + \frac{3}{y}$

$z_x = 2x + y - \frac{3}{x^2}$

$z_y = x + 2y - \frac{3}{y^2}$

$2x + y - \frac{3}{x^2} = 0$

$x + 2y - \frac{3}{y^2} = 0$

СИСТЕМ?

$2x + y - \frac{3}{x^2} - x - 2y + \frac{3}{y^2} = 0$

$x - y - \frac{3}{x^2} + \frac{3}{y^2} = 0$

$\frac{x^3 - y^3 - 3y^2 + 3x^2}{x^2y^2} = 0$

$\frac{(x-y)(x^2+xy+y^2) + 3(x^2-y^2)}{x^2y^2} = 0$ $\quad x, y \neq 0$

$(x-y)\left(x^2+xy+y^2+3(x+y)\right) = 0$

$x - y = 0 \Rightarrow x = y$

$2x + x - \frac{3}{x^2} = 0$

$3x - \frac{3}{x^2} = 0$

$3\left(x - \frac{1}{x^2}\right) = 0$

$x = \frac{1}{x^2}$

$x^3 = 1$

$x = 1$

$y = 1$

$\boxed{M(1,1)}$

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27. $z(x,y) = 3 - x + 6y - y\sqrt{x} - y^2$

$z_x = -1 - \frac{y}{2\sqrt{x}}$

$z_y = 6 - \sqrt{x} - 2y$

$-1 - \frac{y}{2\sqrt{x}} = 0$

$6 - \sqrt{x} - 2y = 0$

$\frac{-y}{2\sqrt{x}} = 1$

$y = -2\sqrt{x}$

$6 - \sqrt{x} + 4\sqrt{x} = 0$

$3\sqrt{x} + 6 = 0$

$3(\sqrt{x} + 2) = 0$

$\sqrt{x} = -2 \perp$

$2y = 6 - \sqrt{x}$

$y = \frac{6 - \sqrt{x}}{2}$

$\frac{\frac{6-\sqrt{x}}{2}}{2\sqrt{x}} = -1$

$\frac{6 - \sqrt{x}}{4\sqrt{x}} = -1$

$6 - \sqrt{x} = -4\sqrt{x}$

$3\sqrt{x} + 6 = 0$

$\sqrt{x} + 2 = 0 \perp$

$\boxed{\phi \text{ ЗА НЕМА ЛОК ЕКСТРЕМУМА}}$

28. $z(x,y) = \frac{1}{x} + \frac{1}{y} + xy$

$z_x = -\frac{1}{x^2} + y$

$z_y = -\frac{1}{y^2} + x$

$y - \frac{1}{x^2} = 0$

$x - \frac{1}{y^2} = 0$

$y = \frac{1}{x^2}$

$x - \frac{1}{\frac{1}{x^4}} = 0$

$x - x^4 = 0$

$x(1 - x^3) = 0$

$x = 0 \quad \vee \quad x^3 = 1$

$y = \frac{1}{0} \qquad x = 1$

$\perp \qquad y = 1$

$\boxed{M(1,1)}$

$$\boxed{\begin{aligned} z_{xx} &= \frac{2}{x^3} \\ z_{yy} &= \frac{2}{y^3} \\ z_{xy} &= 1 \end{aligned}}$$

$M(1,1)$

$r = 2;\ t = 2;\ s = 1;\ \Delta = 3$

$\Delta > 0;\ r > 0 \Rightarrow \boxed{M - \text{лок. мин.}}$

(29) $z(x,y) = (x-y)\, e^{-(x^2+xy+y^2)}$

$Z_x = e^{-(x^2+xy+y^2)} + (x-y)\cdot e^{-(x^2+xy+y^2)}\cdot(-2x-y)$

$= \boxed{e^{-(x^2+xy+y^2)}\left((x-y)(-2x-y)+1\right)}$

$Z_y = -e^{-(x^2+xy+y^2)} + (x-y)\cdot e^{-(x^2+xy+y^2)}(-x-2y)$

$= \boxed{e^{-(x^2+xy+y^2)}\left((x-y)(-x-2y)-1\right)}$

$e^{-(x^2+xy+y^2)} + e^{-(x^2+xy+y^2)}(x-y)(-2x-y) = 0$

$-e^{-(x^2+xy+y^2)} + e^{-(x^2+xy+y^2)}(x-y)(-x-2y) = 0$

$e^{-(x^2+xy+y^2)}\left((x-y)(-2x-y)+(x-y)(-x-2y)\right) = 0$

$(x-y)(-2x-y-x-2y) = 0$

$(x-y)(-3x-3y) = 0$

$-3(x-y)(x+y) = 0$

$\boxed{x=y}$ → не дава се решение

$\boxed{x=-y}$

$(-y-y)(-2y-y)+1) = 0$

$-2y\cdot(-3y)+1 = 0$

$1-6y^2 = 0$

$6y^2 = 1$

$y^2 = \frac{1}{6}$

$y = \pm\frac{1}{\sqrt6}$

$\boxed{M_1\left(\frac{1}{\sqrt6}, -\frac{1}{\sqrt6}\right)}$

$\boxed{M_2\left(-\frac{1}{\sqrt6}, \frac{1}{\sqrt6}\right)}$

$M_1\left(\frac{1}{\sqrt6}, -\frac{1}{\sqrt6}\right)$

$r = e^{-\left(\frac16-\frac16+\frac16\right)}\left(-\frac{2}{\sqrt6}+\frac{1}{\sqrt6}\right)^2\left(\frac{1}{\sqrt6}+\frac{1}{\sqrt6}\right)+1-\frac{4}{\sqrt6}-\frac{1}{\sqrt6}$

$r = e^{-\frac16}\left(\frac{1}{\cancel{6}_3}\cdot\frac{\cancel{2}}{\sqrt6}+1-\frac{5}{\sqrt6}\right) = e^{-\frac16}\left(\frac{1+3\sqrt6-15}{3\sqrt6}\right)$

$Z_{xx} = e^{-(x^2+xy+y^2)}(-2x-y)\left((x-y)(-2x-y)+1\right) + e^{-(x^2+xy+y^2)}(-4x+y)$

$= \boxed{e^{-(x^2+xy+y^2)}\left((-2x-y)^2(x-y)+1-4x+y\right)}$ $2x+y-x+y$

$Z_{yy} = e^{-(x^2+xy+y^2)}(-x-2y)\left((x-y)(-x-2y)-1\right) + e^{-(x^2+xy+y^2)}\left(-1(-2x-y)+(x-y)(-1)\right)$

$= \boxed{e^{-(x^2+xy+y^2)}\left((-x-2y)^2(x-y)-1+x+2y\right)}$

$Z_{xy} = e^{-(x^2+xy+y^2)}(-x-2y)\left((x-y)(-2x-y)+1\right) + e^{-(x^2+xy+y^2)}(2x+y-x+y)$

$= \boxed{e^{-(x^2+xy+y^2)}\left((x+2y)(x-y)(2x+y)+1\right)}$

$\log_a b = x$ $\quad a^0 = 1$

$a^x = b \quad b > 1$

$|x| - |y| > 0 \quad D: (0, +\infty) \quad x > y$

$x - y > 0 \quad (x, y) \in (0, +\infty), x > y$

$x > y \quad (x, y) \in (-\infty, 0) \quad x < y$

$\frac{1}{x - y} \quad x \neq y \quad (x, y) \in ($

$y \neq \pm 3 \quad y^2 - x^2 + 1$

$(x-y)(x+y) + 1$

$y^2 - x^2 \neq -1 \quad (x-y)(x+y) \neq -1$

$y^2 \neq x^2 - 1 \quad x - y = -x - y$

$y \neq \pm\sqrt{x^2 - 1}$

$x^{\frac{1}{2}} \cdot x^{\frac{1}{3}} = x^{\frac{3+2}{6}} \quad x^{\frac{5}{6}} \quad \sqrt[6]{x^5}$

$\left(1 + \frac{1}{f(x)}\right)^{f(x)}$

$\sqrt[3]{h} \cdot \sqrt{6} = \sqrt[6]{}$

$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{\sqrt[n]{n+1}} \quad \sqrt[2]{x} \cdot \sqrt[3]{x} = \sqrt[6]{x^2}$

-1

$\lim_{n \to \infty} \frac{\frac{(-1)^n}{\sqrt[n+1]{n+2}}}{\frac{(-1)^{n-1}}{\sqrt[n]{n+1}}} = \lim_{n\to\infty} \frac{(-1)^n \cdot \sqrt[n]{n+1}}{(-1)^{n-1} \cdot \sqrt[n+1]{n+2}} = \lim (-1) \cdot \sqrt[n(n+1)]{}$

$\sqrt[n+1]{n+2} = \sqrt[n+1]{n+1+1}$

$\frac{\sqrt[n(n+1)]{(n+1)^{(n+1)}}}{\sqrt[n(n+1)]{(n+2)^n}} = \sqrt[n(n+1)]{\frac{(n+1)^{n+1}}{(n+2)^n}}$

$$(2n-1)!! = (2n-1)(2n-3)(2n-5)\cdots 3\cdot 1$$

$$(2n)!! = 2n(2n-2)(2n-4)\cdots 4\cdot 2$$

$$2n(2n-2)!!$$

$$\sum_{n=1}^{\infty} \frac{(2n)!!}{(2n-1)!!}$$

$$\ln xy = \ln x + \ln y$$

$$\cancel{\ln(xy) = \ln x \ln y}$$

$$\lim_{n\to\infty} \frac{\frac{(2n+1)!!}{(2n)!!}}{\frac{(2n)!!}{(2n-1)!!}} = \frac{(2n+1)!!\,(2n-1)!!}{(2n)!!\,(2n)!!} = \frac{(2n+1)(2n-1)!!\,(2n-1)!!}{2^n\cdot n!\cdot 2^n\cdot n!}$$

$R = 1$ $\quad x \in (-\infty, +\infty)$

$x = 1$

$x = -1$

$\frac{1}{n^p}$ $\quad p \le 1$

$p > 1$

$\frac{(-1)^n}{n}$

1) $\lim \frac{1}{n^p} = 0$

2) $f(x) = \frac{1}{x^p}$

$f(x)' = -\frac{1}{x^p}$

KB

KB $\quad x \in (-1, 1)$

$x \in [1 \quad x \in [-1, 1]$

$(2n)!! = 2n(2n-2)(2n-4)$

$(2n+2)!! = (2n+2)(2n)!!$

$(2n+2)\,2^n \cdot n!$

$2^{n+1}(n+1)!$

$2n+1$

<u>Подручје конвергенције</u>. Скуп вредности аргумената x за које ред функција

$$f_1(x) + f_2(x) + \cdots + f_n(x) + \cdots \qquad (1)$$

конвергира, називамо подручјем конвергенције тог реда. Функцију $S(x) = \lim_{n\to\infty} S_n(x)$, где је $S_n = f_1(x) + f_2(x) + \cdots + f_n(x)$, а x припада подручју конвергенције називамо сумом реда, а $R_n(x) = S(x) - S_n(x)$ остатком реда.

У једноставним случајевима је за одређивање полупречника конвергенције реда (1) довољно применити на тај ред познате критеријуме конвергенције, сматрајући x фиксираним.

Пример 1. Одредити подручје конвергенције реда $\frac{x+1}{1\cdot 2} + \frac{(x+1)^2}{2\cdot 2^2} + \frac{(x+1)^3}{3\cdot 2^3} + \cdots + \frac{(x+1)^n}{n\cdot 2^n} + \cdots$

Решење: Означимо са a_n општи члан реда, па ћемо имати:

$$\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = \lim_{n\to\infty} \frac{|x+1|^{n+1}\, 2^n \cdot n}{2^{n+1}(n+1)\,|x+1|^n} = \frac{|x+1|}{2}$$

На основу Даламберовог крит. можемо установити да ред конв. (и то апсолутно) ако је $|x+1|/2 < 1$, тј. за $-3 < x < 1$; ред див. ако је $|x+1|/2 > 1$, тј. ако је $-\infty < x < -3$ или $1 < x < \infty$. За $x = 1$ добијамо хармонијски ред $1 + 1/2 + 1/3 + \cdots$ који див., а за $x = -3$ $-1 + 1/2 - 1/3 + \cdots$ који (према Лајбницовом крит) конвергира (условно). Ред дакле конвергира за $-3 \le x < 1$.

Степени редови

За сваки степени ред

$$c_0 + c_1(x-a) + c_2(x-a)^2 + \cdots + c_n(x-a)^n + \cdots \quad (3)$$

(c_n и a су реални бројеви) постоји такав интервал (интервал конвергенције) $|x-a| < R$ са средиштем у тачки $x=a$, унутар којег ред (3) конвергира апсолутно; за $|x-a| > R$ ред дивергира. Полупречник конвергенције може у спец. случајевима бити и 0 или ∞. У завршним тачкама интервала конвергенције $x = a \pm R$ могућа је како конвергенција тако и дивергенција степеног реда. Интервал конвергенције одређује се обично помоћу Даламберовог или Кошијевог критеријума примењујући их на ред чији су чланови апсолутне вредности чланова задатог реда (3).

Применом на ред апсолутних вредности

$$|c_0| + |c_1||x-a| + \cdots + |c_n||x-a|^n + \cdots$$

Даламберов и Кошијев критеријум конвергенције добијамо за полупречник конвергенције степеног реда (3) одговарајуће формуле:

$$R = \frac{1}{\lim\limits_{n\to\infty} \sqrt[n]{|c_n|}} \quad \text{и} \quad R = \lim_{n\to\infty} \left|\frac{c_n}{c_{n+1}}\right|$$

Но, употреба тих ф-ла захтева опрез због тога што лимеси са десних страна тих формула често не постоје. Тако, нпр. ако има бесконачно много коефицијената c_n који су једнаки нули (то је посебно испуњено у случају када ред има само чланове са парним степенима или само са непарним степенима од $(x-a)$) није могуће применити те ф-ле. Зато се, за одређивање интервала конвергенције препоручује примена Даламберовог или Кошијевог критеријума непосредно, као је то пре учињено приликом разматрања реда (2) где нису употребљене опште ф-ле за полупречник конвергенције.

10. $\sum_{n=1}^{\infty} \frac{1}{n^x}$ — за $x>1$ апс. конв., за $x \le 1$ див.

11. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^x}$ — за $x>1$ апс. конв., за $0<x\le 1$ конв., али не апс., за $x \le 0$ див.

12. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^{\ln x}}$ — за $x>e$ конв. апс., за $1<x\le e$ конв., али не апс., за $x \le 1$ див.

13. $\sum_{n=1}^{\infty} \frac{\sin(2n-1)x}{(2n-1)^2}$ — $-\infty < x < \infty$

14. $\sum_{n=1}^{\infty} 2^n \sin\frac{x}{3^n}$ — $-\infty < x < \infty$

15. $\sum_{n=0}^{\infty} \frac{\cos nx}{e^{nx}}$ $\longrightarrow$ Апс. конв. за $x>0$, див. за $x \le 0$. Решење. 1) $|a_n| \le \frac{1}{e^{nx}}$, а за $x>0$ ред с општим чланом $1/e^{nx}$ конв.; 2) $1/e^{nx} \ge 1$ за $x \le 0$ а $\cos nx$ не тежи к нули ако $n \to \infty$, јер би из $\cos nx \to 0$ следило да $\cos 2nx \to -1$; према томе је за $x \le 0$ нарушен потребан услов конвергенције.

16. $\sum_{n=0}^{\infty} (-1)^{n+1} e^{-n\sin x}$ $\longrightarrow$ Апс. конв. за $2k\pi < x < (2k+1)\pi$ ($k=0, \pm 1, \pm 2, \dots$); у осталим тачкама див.

17. $\sum_{n=1}^{\infty} \frac{n!}{x^n}$ — свуда дивергира

18. $\sum_{n=1}^{\infty} \frac{1}{n!\,x^n}$ — апс. конв. за $x \ne 0$

19. $\sum_{n=1}^{\infty} \frac{1}{(2n-1)x^n}$ — $x>1$, $x \le -1$

20. $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{(x-2)^n}$ — $x>3$, $x<1$

21. $\sum_{n=0}^{\infty} \frac{2n+1}{(n+1)^5 x^{2n}}$ — $x \ge 1$, $x \le -1$

22. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n \cdot 3^n (x-5)^n}$ — $x \ge 5\frac{1}{3}$, $x < 4\frac{2}{3}$

23. $\sum_{n=1}^{\infty} \frac{n^n}{x^{n^n}}$ — $x>1$, $x<-1$

24. $\sum_{n=1}^{\infty} \left(x^n + \frac{1}{2^n x^n}\right)$ — $-1<x<-\frac{1}{2}$, $\frac{1}{2}<x<1$. Упутство: За ове вредности x конвергира како ред $\sum_{k=1}^{\infty} x^k$, тако и ред $\sum_{k=1}^{\infty} \frac{1}{2^k x^k}$. За $|x| \ge 1$ и за $|x| \le \frac{1}{2}$ општи члан реда не тежи нули.

25. $\sum_{n=-1}^{\infty} x^n$ — $-1 < x < 0$, $0 < x < 1$

$z = x^2 - y^2 \qquad 5x^2 - 6xy + 3y^2 = 8$

$L(x,y,\lambda) = x^2 - y^2 + \lambda(5x^2 - 6xy + 3y^2 - 8) = 0$

$L_x = 2x + 10\lambda x - 6\lambda y \quad / : 2$

$L_y = -2y - 6\lambda x + 10\lambda y \quad / : 2$

$L_\lambda = 5x^2 - 6xy + 3y^2 - 8$

$2x + 10\lambda x - 6\lambda y = 0$

$-2y - 6\lambda x + 10\lambda y = 0$

$x + 5\lambda x - 3\lambda y = 0 \qquad 5+\lambda$

$-y - 3\lambda x + 5\lambda y = 0$

$x(5+\lambda) - 3\lambda y = 0$

$-3\lambda x - y(5\lambda + 1) = 0$

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1. $z = x^2 - y^2$ $\quad 5x^2 - 6xy + 5y^2 = 8$

$L(x,y,\lambda) = x^2 - y^2 + \lambda(5x^2 - 6xy + 5y^2 - 8)$

$L_x = 2x + 10\lambda x - 6\lambda y$,

$L_y = -2y - 6\lambda x + 10\lambda y$

$L_\lambda = 5x^2 - 6xy + 5y^2 - 8$

$x + 5\lambda x - 3\lambda y = 0$

$-y - 3\lambda x + 5\lambda y = 0$

$x(1+5\lambda) - 3\lambda y = 0$

$-3\lambda x + y(5\lambda - 1) = 0$

$\Delta = \begin{vmatrix} 1+5\lambda & -3\lambda \\ -3\lambda & 5\lambda - 1 \end{vmatrix} = 25\lambda^2 - 1 - 9\lambda^2 = 16\lambda^2 - 1$

$\Delta = 0 \Rightarrow 16\lambda^2 - 1 = 0$

$16\lambda^2 = 1$

$\lambda^2 = \frac{1}{16} \Rightarrow \boxed{\lambda = \pm\frac{1}{4}}$

$\lambda = \frac{1}{4} \rightarrow \frac{9}{4}x - \frac{3}{4}y = 0 \Rightarrow y = 3x$

$5x^2 - 6x\cdot 3x + 5\cdot 9x^2 - 8 = 0$

$50x^2 - 18x^2 - 8 = 0$

$32x^2 = 8$

$4x^2 = 1$

$x^2 = \frac{1}{4} \Rightarrow x = \pm\frac{1}{2}$

$\boxed{M_1\left(\frac{1}{2}, \frac{3}{2}\right)}$ $\lambda = \frac{1}{4}$

$\boxed{M_2\left(-\frac{1}{2}, -\frac{3}{2}\right)}$ $\lambda = \frac{1}{4}$

$\lambda = -\frac{1}{4} \rightarrow -\frac{1}{4}x + \frac{3}{4}y = 0 \Rightarrow 3y = x$

$\Rightarrow y = \pm\frac{1}{2}$

$\boxed{M_3\left(\frac{3}{2}, \frac{1}{2}\right)}$ $\lambda = -\frac{1}{4}$

$\boxed{M_4\left(\frac{3}{2}, -\frac{1}{2}\right)}$ $\lambda = -\frac{1}{4}$

$L_{xx} = 2 + 10\lambda$

$L_{yy} = -2 + 10\lambda$

$L_{xy} = -6\lambda$

$d^2L = (2+10\lambda)dx^2 - 12\lambda\, dx\, dy + (10\lambda - 2)dy^2$

$d^2L\left(\lambda = \frac{1}{4}\right) = \frac{9}{2}dx^2 - 3dx\,dy + \frac{1}{2}dy^2$

$5x^2 - 6xy + 5y^2 - 8 = 0$

$\boxed{(10x - 6y)dx + (10y - 6x)dy = 0}$

$M_1\left(\frac{1}{2}, \frac{3}{2}\right) \Rightarrow -4dx + 12dy = 0 \Rightarrow \boxed{dx = 3dy}$ $\quad M_1 \rightarrow$ мин.

$d^2L(M_1) = \frac{9}{2}\cdot 9dy^2 - 3\cdot 3dy\,dy + \frac{1}{2}dy^2 = \frac{81}{2}dy^2 - 9dy^2 + \frac{1}{2}dy^2 = 32dy^2 \geq 0$

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$M_2\left(-\frac{1}{2}, -\frac{3}{2}\right)$ $(-5+9)dx + (-15+3)dy = 0$

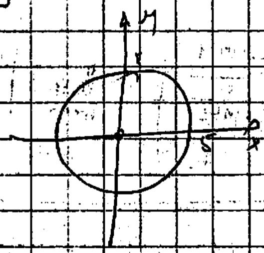
$4dx - 12dy = 0$ $\boxed{dx = 3dy}$

$\Rightarrow$ $\boxed{M_2 - e.\ MAX}$

$z = x^2+y^2-12x+16y$ $D = \{(x,y) \in \mathbb{R}^2 \mid x^2+y^2 \leq 25\}$

$\text{int}\,D = \{x,y \mid x^2+y^2 < 25\}$

$\partial D = \{x,y \mid x^2+y^2 = 25\}$



$z_x = 2x - 12$ $x = 6$

$z_y = 2y + 16$ $y = -8$

$(6,-8) \notin \text{int}\,D$

$L(x,y,\lambda) = x^2+y^2-12x+16y + \lambda(x^2+y^2-25)$

$L_x = 2x - 12 + 2\lambda x = x + \lambda x - 6 = x(1+\lambda) - 6 = 0$

$L_y = 2y + 16 + 2y\lambda = y + 4 + y\lambda = y(1+\lambda) + 4 = 0$

$L_\lambda = x^2+y^2-25$

$x(1+\lambda) - 6 = 0$ $\Rightarrow x = \frac{6}{1+\lambda}$

$y(1+\lambda) + 4 = 0$ $y = -\frac{4}{1+\lambda}$

$x^2+y^2-25 = 0$

$\frac{36}{(1+\lambda)^2} + \frac{16}{(1+\lambda)^2} - 25 = 0$

$\frac{52}{(1+\lambda)^2} = 25$

$(1+\lambda)^2 = \frac{52}{25}$

(12) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^{\ln x}}$

$R = \lim_{n\to\infty} \left| \frac{\frac{(-1)^{n+1}}{n^{\ln x}}}{\frac{(-1)^{n+2}}{(n+1)^{\ln x}}} \right| = \lim \left| \frac{(-1)^{n+1} \cdot (n+1)^{\ln x}}{(-1)^{n+2} \cdot n^{\ln x}} \right| = \lim_{n\to\infty} \left| \frac{(-1)(n+1)^{\ln x}}{n^{\ln x}} \right| =$

$= \lim_{n\to\infty} \left| (-1) \cdot \left(\frac{n+1}{n}\right)^{\ln x} \right| = \left| \left(1+\frac{1}{n}\right)^{n \cdot \frac{\ln x}{n}} \right| = e^{\lim \ln \frac{x}{n}}$

$R = \lim_{n\to\infty} \left| \frac{\frac{(-1)^{n+1}}{n}}{\frac{(-1)^{n+2}}{n+1}} \right| = \lim_{n\to\infty} \left| \frac{(-1)^{n+1}(n+1)}{(-1)^{n+2} \cdot n} \right| = \lim_{n\to\infty} \left| \left(1+\frac{1}{n}\right)^{n \cdot \frac{1}{n}} \right| = e^{\frac{1}{n}} = 1$

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(29) $u = \frac{x^4}{12} - \frac{1}{6}x^3(y+z) + \frac{1}{2}x^2 yz + f(y-x, z-x)$

$u_x + u_y + u_z = xyz$

$u_x = \frac{x^3}{3} - \frac{1}{2}x^2(y+z) + xyz + f_u(-1) + f_v(-1)$

$u_y = -\frac{1}{6}x^3 + \frac{1}{2}x^2 z + f_u$

$u_z = -\frac{1}{6}x^3 + \frac{1}{2}x^2 y + f_v$

$u_x + u_y + u_z = \cancel{\frac{x^3}{3}} - \cancel{\frac{1}{2}x^2 y} - \cancel{\frac{1}{2}x^2 z} + xyz - \cancel{f_u} - \cancel{f_v} - \cancel{\frac{1}{6}x^3} + \cancel{\frac{1}{2}x^2 z} + \cancel{f_u} - \cancel{\frac{1}{6}x^3} + \cancel{\frac{1}{2}x^2 y} + \cancel{f_v}$

$\boxed{u_x + u_y + u_z = xyz}$ ✓

$F\left(x + \frac{z}{y}, y + \frac{z}{x}\right) = 0 \qquad x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} + xy = z$

$u = x + \frac{z}{y} \qquad u_x = 1 \qquad u_y = -\frac{z}{y^2}$

$v = y + \frac{z}{x} \qquad v_x = -\frac{z}{x^2} \qquad v_y = 1$

$F_x = 0$

$z_x = F_u u_x + F_v v_x = F_u - \frac{z}{x^2}F_v$

$z_y = F_u u_y + F_v v_y = -\frac{z}{y^2}F_u + F_v$

$xF_u - \frac{z}{x}F_v - \frac{z}{y}F_u + yF_v + xy = z$

$F_u\left(x - \frac{z}{y}\right) + F_v\left(y - \frac{z}{x}\right) + xy = z$

$F_u \cdot \frac{xy-z}{y} + F_v \frac{xy-z}{x} + xy = z$

$(xy-z)(xF_u + yF_v)$

$d^2L\left(\lambda = \frac{1}{4}\right) = \frac{9}{2}dx^2 - \frac{?}{?}$

$5x^2 - 6xy + 5y^2 - 8 = 0$

$\boxed{(10x - 6y)dx + (10y - 6x}$

$M_1\left(\frac{1}{2}, \frac{3}{2}\right) \Rightarrow -4dx +$

$d^2L(M_1) = \frac{9}{2} \; 9dy^2 -$

60) $F\left(x+\frac{z}{y},\ y+\frac{z}{x}\right)=0 \qquad x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}+xy=z$

$u=x+\frac{z}{y} \qquad u_x=1+\frac{z_x}{y} \qquad u_y=\frac{y\cdot z_y-z}{y^2}$

$v=y+\frac{z}{x} \qquad v_x=\frac{x z_x-z}{x^2} \qquad v_y=1+\frac{z_y}{x}$

$F_x=0$

$F_u u_x+F_v v_x=0 \qquad\qquad F_u u_y+F_v v_y=0$

$F_u\cdot\left(1+\frac{z_x}{y}\right)+F_v\cdot\frac{x z_x-z}{x^2}=0 \qquad\qquad F_u\,\frac{y\cdot z_y-z}{y^2}+F_v\cdot\left(1+\frac{z_y}{x}\right)=0$

$F_u+z_x\cdot\frac{F_u}{y}+z_x\cdot\frac{F_v}{x}-\frac{zF_v}{x^2}=0 \qquad\qquad z_y\cdot\frac{F_u}{y}-\frac{F_u z}{y^2}+F_v+z_y\cdot\frac{F_v}{x}=0$

$z_x\left(\frac{F_u}{y}+\frac{F_v}{x}\right)=\frac{zF_v}{x^2}-F_u \qquad\qquad z_y\left(\frac{F_u}{y}+\frac{\cancel{z}}{\cancel{y}}\,\frac{F_v}{x}\right)=\frac{F_u z}{y^2}-F_v$

$z_x\,\frac{F_u\cdot x+F_v\cdot y}{xy}=\frac{zF_v-F_u x^2}{x^2} \qquad\qquad z_y\cdot\frac{F_u\cdot x+F_v\cdot y}{xy}=\frac{F_u z-F_v\cdot y^2}{y^2}$

$z_x=\frac{\cancel{x}y\,(zF_v-F_u x^2)}{x^{\cancel{2}}(F_u x+F_v y)} \qquad\qquad z_y=\frac{x\cancel{y}\,(F_u z-F_v y^2)}{y^{\cancel{2}}(F_u x+F_v y)}$

$z_x=\frac{y\,(zF_v-F_u x^2)}{x\,(F_u x+F_v y)} \qquad\qquad z_y=\frac{x\,(F_u z-F_v y^2)}{y\,(F_u x+F_v y)}$

$\cancel{x}\cdot\frac{y\,(zF_v-F_u x^2)}{\cancel{x}\,(F_u x+F_v y)}+\cancel{y}\cdot\frac{x\,(F_u z-F_v y^2)}{\cancel{y}\,(F_u x+F_v y)}+xy=z$

$\frac{yz\cdot F_v-x^2yF_u+xzF_u-xy^2F_v}{F_u x+F_v y}+xy=z$

$\frac{yF_v(z-xy)+xF_u(z-xy)}{F_u x+F_v y}+xy=z$

$\frac{(z-xy)\cancel{(xF_u+yF_v)}}{\cancel{F_u x+F_v y}}+xy=z$

$z-\cancel{xy}+\cancel{xy}=z$

$z=z$ T

Условни

(21.) $u = x^2+y^2+z^2 \qquad 9x^2+16y^2+36z^2=144$

$L(x,y,z,\lambda) = x^2+y^2+z^2+\lambda(9x^2+16y^2+36z^2-144)$

$L_x = 2x+18x\lambda = 2x(1+9\lambda) \qquad x(1+9\lambda)=0 \Rightarrow \lambda_1 = -\frac{1}{9}$

$L_y = 2y+32y\lambda = 2y(1+16\lambda) \qquad y(1+16\lambda)=0 \Rightarrow \lambda_2 = -\frac{1}{16}$

$L_z = 2z+72z\lambda = 2z(1+36\lambda) \qquad z(1+36\lambda)=0 \Rightarrow \lambda_3 = -\frac{1}{36}$

$L_\lambda = 9x^2+16y^2+36z^2-144$

$1^\circ \quad x=y=z=0 \qquad 0 = -144 \perp$

$2^\circ \quad \lambda = -\frac{1}{9} \qquad 2y\left(1-\frac{16}{9}\right) = 2y\left(\frac{9-16}{9}\right) = -\frac{14}{9}y = 0 \Rightarrow y=0$

$2z\left(1+36\left(-\frac{1}{9}\right)\right)=0 \Rightarrow z=0$

$9x^2-144=0$

$9x^2 = 144 \Rightarrow x^2 = \frac{144}{9} \qquad x = \pm\sqrt{\frac{144}{9}} = \pm\frac{12}{3} = \pm 4$

$\boxed{\lambda_1 = -\frac{1}{9}}\ \boxed{M_1(4,0,0)}\ \boxed{M_2(-4,0,0)}$

$3^\circ \quad \lambda_2 = -\frac{1}{16} \qquad 2x\left(1+9\left(-\frac{1}{16}\right)\right)=0 \Rightarrow x=0$

$2z\left(1+36\left(-\frac{1}{16}\right)\right)=0 \Rightarrow z=0$

$16y^2-144=0 \Rightarrow y^2 = \pm\sqrt{\frac{144}{16}} \Rightarrow y = \pm\frac{12}{4} = \pm 3$

$\boxed{\lambda = -\frac{1}{16}}\ \boxed{M_3(0,3,0)}\ \boxed{M_4(0,-3,0)}$

$4^\circ \quad \lambda_3 = -\frac{1}{36} \qquad 2x\left(1+9\left(-\frac{1}{36}\right)\right)=0 \Rightarrow x=0$

$2y\left(1+16\left(-\frac{1}{36}\right)\right)=0 \Rightarrow y=0$

$36z^2-144=0 \Rightarrow z^2 = \pm\sqrt{\frac{144}{36}} \Rightarrow z = \pm\frac{12}{6} = \pm 2$

$\boxed{\lambda = -\frac{1}{36}}\ \boxed{M_5(0,0,2)}\ \boxed{M_6(0,0,-2)}$

$L_{xx} = 2+18\lambda \qquad L_{xy} = 0$

$L_{yy} = 2+32\lambda \qquad L_{xz} = 0$

$L_{zz} = 2+72\lambda \qquad L_{yz} = 0$

$d^2L = 2(1+9\lambda)dx^2 + 2(1+16\lambda)dy^2 + 2(1+36\lambda)dz^2$

$9x^2+16y^2+36z^2-144=0$

$18x\,dx + 32y\,dy + 72z\,dz = 0$ Аибовреуно

$\lambda = -\frac{1}{9}$ $\quad 2\left(1-\frac{16}{9}\right)dy^2 + 2\left(1-\frac{36}{9}\right)dz^2 = d^2L$

$$\boxed{d^2L = -\frac{14}{9}dy^2 - 6dz^2}$$

$d^2L < 0$ $\quad \left.\begin{matrix} M_1(4,0,0) \\ M_2(-4,0,0) \end{matrix}\right\}$ максимум

$\lambda = -\frac{1}{16}$ $\quad 2\left(1-\frac{9}{16}\right)dx^2 + 2\left(1-\frac{36}{16}\right)dz^2 = d^2L$

$d^2L = \frac{14}{9}dx^2 - \frac{40}{16}dz^2 = \frac{14}{9}dx^2 - \frac{5}{2}dz^2$ $\quad dy = 0$

$2y + 2z + yz\lambda = 0$

$2y + z(2 + y\lambda) = 0$

$z = -\frac{2y}{2+y\lambda}$

$2x + 2z + xz\lambda = 0$

$2x - \frac{4y}{2+y\lambda} - \frac{2xy}{2+y\lambda} = 0$

$$\boxed{\begin{matrix} 2x - \frac{4y+2xy}{2+y\lambda} = 0 \\ 2x + 2y + xy\lambda = 0 \end{matrix}}$$

$A = f''_{xx}(a,b)$

$B = f_{xy}(a,b)$ $\quad \Delta = AC - B^2$

$C = f_{xy}(a,b)$

$V = abc \quad = xyz = p$

$2(ab + ac + bc) = U$

$U = 2(xy + xz + yz)$

$L(x,y,z,\lambda) = 2xy + 2xz + 2yz + \lambda(xyz - p)$

$L_x = 2y + 2z + yz\lambda = 0$ $\quad 2y + 2z + yz\lambda = 0$

$L_y = 2x + 2z + xz\lambda = 0$ $\quad 2x + 2z + xz\lambda = 0$

$L_z = 2x + 2y + xy\lambda = 0$ $\quad 2x + 2y + xy\lambda = 0$

$L_\lambda = xyz - p \sim$

$$\begin{vmatrix} 0 & 2+z\lambda & 2+y\lambda \\ 2+z\lambda & 0 & 2+x\lambda \\ 2+y\lambda & 2+x\lambda & 0 \end{vmatrix} = -(2+z\lambda)\begin{vmatrix} 2+z\lambda & 2+y\lambda \\ 2+x\lambda & 0 \end{vmatrix} + (2+y\lambda)\begin{vmatrix} 2+z\lambda & 2+y\lambda \\ 0 & 2+x\lambda \end{vmatrix}$$

$\Delta = -(2+z\lambda)\left(-(2+x\lambda)(2+y\lambda)\right) + (2+y\lambda)\left((2+z\lambda)(2+x\lambda)\right)$

$\Delta = 2(2+z\lambda)(2+x\lambda)(2+y\lambda)$

$\boxed{\lambda_1 = -\frac{2}{z}}$ $\quad \boxed{\lambda_2 = -\frac{2}{x}}$ $\quad \boxed{\lambda_3 = -\frac{2}{y}}$ $\quad x, y, z \neq 0$

$2y + 2z - 2y = 0 \Rightarrow z = 0$

$\cancel{2x} + 2z - 2x = 0 \Rightarrow z = 0$

$2x + 2y - \frac{xy}{z} = 0$

$$2x - \frac{4y+2xy}{2+y\lambda} = 0$$

$$2x+2y+xy\lambda = 0$$

$$\frac{2x(2+y\lambda)-4y-2xy}{2+y\lambda} = 0$$

$$4x + 2xy\lambda - 4y - 2xy = 0$$

$$2x+2y+xy\lambda = 0$$

$$x+y=a$$

$$x^2+y^2=p$$

(46.) $u = x^2+y^2$ $\quad x+y=a$ – jewb

$$\boxed{M\left(\frac{a}{2}, \frac{a}{2}\right)}$$

(47.) $u = x^3+y^3 \quad x+y=a$

$$L(x,y,\lambda) = x^3+y^3+\lambda(x+y-a)$$

$L_x = 3x^2+\lambda \qquad 3x^2+\lambda = 0 \Rightarrow \lambda_1 = -3x^2$

$L_y = 3y^2+\lambda \qquad 3y^2+\lambda = 0 \Rightarrow \lambda_2 = -3y^2$

$L_\lambda = x+y-a$

$\lambda_1 = -3x^2 \qquad 3y^2-3x^2 = 0$

$3y^2 = 3x^2$

$\boxed{y = \pm x}$

1° $y = x$

$2x - a = 0 \quad x = \frac{a}{2} \quad M_1\left(\frac{a}{2}, \frac{a}{2}\right)$

2° $y = -x$

$-a = 0 \; \bot$

$\lambda_2 = -3y^2 \qquad 3x^2 - 3y^2 = 0$

$3x^2 = 3y^2$

$\boxed{x = \pm y}$

1° $x = y$

$2y - a = 0 \quad y = \frac{a}{2} \quad M\left(\frac{a}{2}, \frac{a}{2}\right)$

2° $x = -y$

$-a = 0 \; \bot$

$$\boxed{\lambda = -\frac{3a^2}{4} \quad M\left(\frac{a}{2}, \frac{a}{2}\right)}$$

$L_{xx} = 6x$

$L_{yy} = 6y \qquad d^2L = 6x\,dx^2 + 6y\,dy^2$

$L_{xy} = 0 \qquad d^2L = 3a\,dx^2 + 3a\,dy^2 > 0 \quad M$ je мин.

$$u_{мин} = \frac{a^3}{8} + \frac{a^3}{8} = \frac{2a^3}{8} = \boxed{\frac{a^3}{4}}$$

(79) $u = x^2+y^2+z^2 \qquad x+y+z=a$

$L(x,y,z,\lambda) = x^2+y^2+z^2 + \lambda(x+y+z-a)$

$L_x = 2x+\lambda \qquad 2x+\lambda=0 \Rightarrow \lambda=-2x$

$L_y = 2y+\lambda \qquad 2y+\lambda=0 \qquad -2x+2y=0 \Rightarrow x=y$

$L_z = 2z+\lambda \qquad 2z+\lambda=0 \qquad -2x+2z=0 \Rightarrow x=z$

$L_\lambda = x+y+z-a \qquad x+y+z-a=0$

$3x-a=0$

$x=\frac{a}{3} \qquad M\left(\frac{a}{3},\frac{a}{3},\frac{a}{3}\right) \qquad \lambda=-\frac{2a}{3}$

$L_{xx}=2 \qquad L_{xy}=0$

$L_{yy}=2 \qquad L_{xz}=0 \qquad d^2L = 2dx^2+2dy^2+2dz^2>0$

$L_{zz}=2 \qquad L_{yz}=0$

M је мин.

$u_{мин} = \frac{3a^2}{9} = \boxed{\frac{a^2}{3}}$

(80) $u=+\sqrt{x^2+y^2} \qquad P=\frac{xy}{2} \Rightarrow xy-2p=0$

$L(x,y,\lambda) = \sqrt{x^2+y^2} + \lambda(xy-2p)$

$L_x = \frac{x}{\sqrt{x^2+y^2}} + \lambda y \qquad \frac{x}{\sqrt{x^2+y^2}}+\lambda y=0 \Rightarrow \frac{x+\lambda y\sqrt{x^2+y^2}}{\sqrt{x^2+y^2}}=0$

$L_y = \frac{y}{\sqrt{x^2+y^2}} + \lambda x \qquad \frac{y}{\sqrt{x^2+y^2}}+\lambda x=0 \Rightarrow \frac{y+\lambda x\sqrt{x^2+y^2}}{\sqrt{x^2+y^2}}=0$

$L_\lambda = xy-2p$

$x+\lambda y\sqrt{x^2+y^2}=0$

$y+\lambda x\sqrt{x^2+y^2}=0$

$L_{xx} = \sqrt{x^2+y^2} - \frac{x^2}{\sqrt{x^2+y^2}} = \frac{y^2}{\sqrt{x^2+y^2}}$

$\lambda y\sqrt{x^2+y^2} - \lambda x\sqrt{x^2+y^2} + x - y = 0$

$L_{yy} = \sqrt{x^2+y^2} - \frac{y^2}{\sqrt{x^2+y^2}} = \frac{x^2}{\sqrt{x^2+y^2}}$

$\lambda\sqrt{x^2+y^2}(y-x) - (y-x) = 0$

$(y-x)(\lambda\sqrt{x^2+y^2}-1)=0$

$L_{xy} = -\frac{x}{x^2+y^2} + \lambda$

1° $y=x$

$x^2=2p \qquad x=\pm\sqrt{2p} \qquad \boxed{M_1(\sqrt{2p},\sqrt{2p}),\ M_2(-\sqrt{2p},-\sqrt{2p})}$

$\frac{x}{\sqrt{x^2+y^2}}+\lambda y=0$

$\lambda y = \frac{x}{\sqrt{x^2+y^2}} \Rightarrow \boxed{\lambda=\frac{x}{y\sqrt{x^2+y^2}}}$

2° $\lambda\sqrt{x^2+y^2}=1$

$\lambda=\frac{1}{\sqrt{x^2+y^2}} \qquad x=-y$

$-x^2=2p$

$\frac{x+y}{\sqrt{x^2+y^2}}=0 \qquad x^2=-2p \qquad p>0 \quad \perp$

$x+y=0$

$$d^2L = \frac{y^2}{\sqrt{x^2+y^2}}dx^2 + 2\left(\lambda - \frac{x}{x^2+y^2}\right)dxdy + \frac{x^2}{\sqrt{x^2+y^2}}dy^2$$

$$\lambda = \frac{x}{y\sqrt{x^2+y^2}} \qquad M_1(\sqrt{2p}, \sqrt{2p}) \quad M_2(-\sqrt{2p}, -\sqrt{2p})$$

$$d^2L = \frac{y^2}{\sqrt{x^2+y^2}}dx^2 + 2\left(\frac{x-xy}{y\sqrt{x^2+y^2}}\right)dxdy + \frac{x^2}{\sqrt{x^2+y^2}}dy^2$$

$$d^2L = \frac{2p}{\sqrt{2p+2p}}dx^2 + 2\left(\frac{\sqrt{2p}-2p}{\sqrt{2p}\sqrt{2p+2p}}\right)dxdy + \frac{2p}{\sqrt{4p}}dy^2$$

$$= \frac{\cancel{2}p}{\cancel{2}\sqrt{p}}dx^2 + \frac{\cancel{2}\cancel{\sqrt{2p}}(1-\sqrt{2p})}{\cancel{\sqrt{2p}}\cdot 2\sqrt{p}}dxdy + \frac{2p}{2\sqrt{p}}dy^2$$

$$= \sqrt{p}\,dx^2 + (1-\sqrt{2p})dxdy + \sqrt{p}\,dy^2$$

$1°$ $dx = dy$ $\quad 2\sqrt{p}\,dx^2 + dx^2 - 2\sqrt{2p}\,dx^2 = dx^2(2\sqrt{p} - 2\sqrt{2p} + 1)$

$$= dx^2(\sqrt{p}(2 - 2\sqrt{2} + 1)$$

$$= dx^2 \cdot \sqrt{p} \cdot (3 - 2\sqrt{2}) > 0$$

$2°$ $dx = -dy$ $\quad 2\sqrt{p}\,dx^2 - (1-\sqrt{2p})dx^2 = dx^2(2\sqrt{p} - 1 + \sqrt{2p}) =$

$$= dx^2(\sqrt{p}(2 - 1 + \sqrt{2})$$

$$= dx^2\sqrt{p}\cdot(1+\sqrt{2}) > 0$$

$\boxed{M(\sqrt{2p}, \sqrt{2p})} \to$ мин

$\cancel{xp}$ $xy - 2p = 0$

$y\,dx + x\,dy = 0$

$y\,dx = -x\,dy$

$dx = -\frac{x\,dy}{y} = -dy$

$$u_{мин} = \sqrt{2p+2p} = \boxed{2\sqrt{p}}$$

(51) $u = xyz \qquad P = 2(xy + xz + yz)$

(52) $P = 2(xy+xz+yz) \qquad V = xyz$

(53) $u = xyz \qquad \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

(54) $u = \sqrt{x^2+y^2} \qquad \frac{(x-1)^2}{9} + \frac{y^2}{4} = 1$

(55) $13x^2 + 13y^2 + 10xy - 72 = 0$

$u = \sqrt{x^2+y^2}$

56. $(x+y)^2+4y=0$

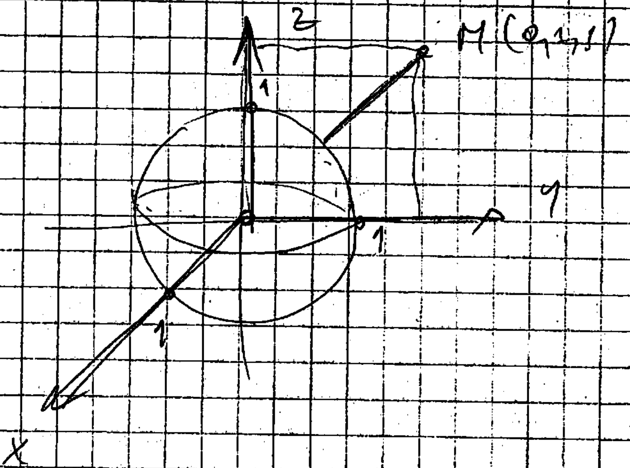
$3x-6y+4=0$

$$z=\frac{|3x-6y+4|}{\sqrt{9+36}}$$

$z=(3x-6y+4)^2$ услов $(x+y)^2+4y=0$

57. $u=(x+2y+3z)^2$ $x^2+2y^2+3z^2=0$

58.



$x+y+z=1$

$u=x^2+y^2+z^2$ условия: $x^2+y^2+z^2=1$

$\boxed{u=x^2+(y-3)^2+(z-3)^2}$ $x+y+z=1$

59. $u=x+y$ услов $P=xy$

60. $u=xy$ $S=x+y$

61. $u=x+y+z$ услов: $xyz=P$

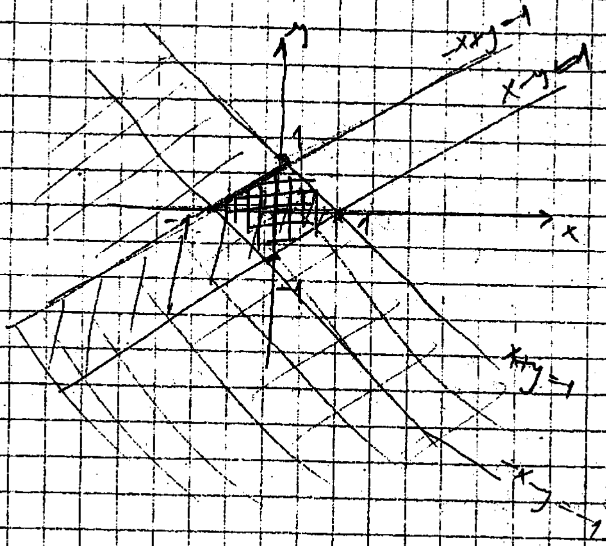
62. $u=[x^2+(y-1)^2+(z-1)^2]^2+[(x-1)^2+(y-2)^2+(z+1)^2]^2$ услов: $x+y+z=1$

~~Најмања~~ и ~~највећа~~

(1) $z = x^2 + 2xy - 3y^2 + y$ $\qquad D = \{(x,y) \in R^2 \mid |x| + |y| \le 1\}$

$|x| + |y| \le 1$

$x + y \le 1$
$-x + y \le 1$
$x - y \le 1$
$-x - y \le 1$



$z = x^2 + 2xy - 3y^2 + y$

I) $z_x = 2x + 2y$ $\qquad x + y = 0$

$z_y = 2x - 6y + 1$ $\qquad 2x - 6y + 1 = 0$

$x = -y$

$-2y - 6y + 1 = 0$

$-8y = -1 \Rightarrow y = \frac{1}{8}$ $\quad x = -\frac{1}{8}$ $\quad M\left(-\frac{1}{8}, \frac{1}{8}\right) \in \text{int} D$

II) 1° $y = 1 - x$ $\quad 0 \le x \le 1$; $\quad 0 \le y \le 1$

$z(x,y) = z(x, 1-x) = x^2 + 2x(1-x) - 3(1-x)^2 + 1 - x$

$= x^2 + 2x - 2x^2 - 3 + 6x - 3x^2 + 1 - x$

$= -4x^2 + 7x - 2$

$\varphi(x) = -4x^2 + 7x - 2$

$\varphi'(x) = -8x + 7 = 0$

$-8x = -7 \Rightarrow x = \frac{7}{8}$ $\qquad \boxed{M_1\left(\frac{7}{8}, \frac{1}{8}\right)}$

$y = \frac{1}{8}$

2° $y = 1 + x$ $\quad -1 \le x \le 0$; $\quad 0 \le y \le 1$

$z(x,y) = z(x, 1+x) = x^2 + 2x(1+x) - 3(1+x)^2 + 1 + x$

$= x^2 + 2x + 2x^2 - 3 - 6x - 6x^2 + 1 + x$

$= -3x^2 - 3x - 2$

$\varphi(x) = 3x^2 - 3x - 2$ $\qquad 6x - 3 = 0$

$\qquad x = \frac{1}{2}$ $\qquad M\left(\frac{1}{2}, \frac{3}{2}\right) \notin \text{int} D$

$\varphi'(x) = 6x - 3 = 0$ $\qquad y = \frac{3}{2}$

$y = x-1 \quad 0 \le x < 1; \quad -1 \le y < 0$

$z(x,y) = z(x, x-1) = x^2 + 2x(x-1) - 3(x-1)^2 + x - 1$

$= x^2 + 2x^2 - 2x - 3x^2 + 6x - 3 + x - 1$

$= 5x - 4$

$\varphi(x) = 5x - 4$

$\varphi'(x) = 5 \neq 0$

$\left(-\frac{3}{14}, \frac{21}{14}\right) \notin M(D)$

2.2 Області конвергенції

(3.) $\sum_{n=1}^{\infty} (-1)^{n+1} 3^n x^{n+1}$

$|x| < \frac{1}{3}$ абс. конв.

$R = \lim_{n\to\infty} \left| \frac{(-1)^{n+1} 3^n}{(-1)^{n+2} 3^{n+1}} \right| = \lim_{n\to\infty} \left| (-1) \cdot 3^{-1} \right| = \frac{1}{3}$

$|x| > \frac{1}{3}$ див.

$\underline{x = \frac{1}{3}} \quad \sum_{n=1}^{\infty} (-1)^{n+1} \frac{3^n}{3^{n+1}} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{3} = -\frac{1}{3}$ див.

$\underline{x = -\frac{1}{3}} \quad \sum_{n=1}^{\infty} (-1)^{n+1} \cdot \frac{3^n}{(-3)^{n+1}} = \sum_{n=1}^{\infty} \frac{\cancel{(-1)^{n+1}} 3^n}{\cancel{(-1)^{n+1}} \cdot 3^{n+1}} = \frac{1}{3}$ див

$x \in \left(-\frac{1}{3}, \frac{1}{3}\right)$

(4.) $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n(n+1)}$

$R = \lim_{n\to\infty} \left| \frac{\frac{(-1)^n}{n(n+1)}}{\frac{(-1)^{n+1}}{(n+1)(n+2)}} \right| = \lim_{n\to\infty} \left| \frac{(-1)^n \cancel{(n+1)}(n+2)}{(-1)^{n+1} n \cancel{(n+1)}} \right| = \left| \frac{(-1)(n+2)}{n} \right| = 1$

$\underline{x = 1} \quad \sum_{n=1}^{\infty} \frac{(-1)^n}{n(n+1)} \quad \lim_{n\to\infty} \frac{(-1)^n}{n(n+1)} = \lim_{n\to\infty} \frac{1}{n(n+1)} = \frac{1}{\infty} = 0$

$\frac{a_n}{a_{n+1}} = \lim_{n\to\infty} \left(-\frac{n+2}{n}\right) = -1$

(25.) $\sum_{n=2}^{\infty} \left(\frac{n^2+1}{n^2-1}\right)^3 x^n$

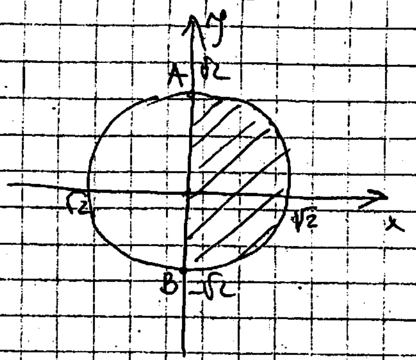
$R = \lim_{n\to\infty} \frac{n^2+1}{n^2-1} = 1$

$\underline{x = 1} \quad \sum_{n=2} \left(\frac{n^2+1}{n^2-1}\right)^3 \quad \lim_{n\to\infty} \left(\frac{n^2+1}{n^2-1}\right)^3 = 1$ див.

$\underline{x = -1} \quad \sum_{n=2}^{\infty} \left(\frac{n^2+1}{n^2-1}\right)^3 (-1)^n \quad a_n \not\to 0 \Rightarrow$ див

$x \in (-1, 1)$

② $z(x,y) = x^2+y^2-(x-y)^3 \qquad D=\{(x,y)\in\mathbb{R}^2 \mid x^2+y^2\le 2,\ x\ge 0\}$



I) унутрашност

$z_x = 2x-3(x-y)^2 \qquad 2x-3(x-y)^2=0$

$z_y = 2y+3(x-y)^2 \qquad \underline{2y+3(x-y)^2=0}$

$2x+2y=0$

$2(x+y)=0 \Rightarrow \underline{x=-y}$

$-2y-3(-2y)^2=0$

$-2y-12y^2=0 \;/ : (-2)$

$y+6y^2=0$

$y(1+6y)=0 \Rightarrow y=0 \quad \vee \quad y=-\frac{1}{6}$

$x=0 \qquad x=\frac{1}{6}$

$\boxed{M_1(0,0)} \qquad \boxed{M_2\left(\frac{1}{6},-\frac{1}{6}\right)}$

$M_1(0,0)\notin \operatorname{int} D$

$\boxed{M_2\left(\frac{1}{6},-\frac{1}{6}\right)\in \operatorname{int} D}$

II) руб

$\overline{AB} \qquad x=0\ ;\ -\sqrt{2}<y<\sqrt{2}$

$z(x,y)=z(0,y)=y^2+y^3=y^2(1+y)$

$\varphi(y)=y^2(1+y)$

$\varphi'(y)=2y(1+y)+y^2=0$

$\varphi'(y)=2y+2y^2+y^2=3y^2+2y=y(3y+2)=0$

$y=0 \quad \vee \quad y=-\frac{2}{3}$

$x=0 \qquad x=0$

$\boxed{M_3(0,0)} \qquad \boxed{M_4\left(0,-\frac{2}{3}\right)}$

$\boxed{M_3(0,0)\in \operatorname{int} D}$

$\boxed{M_4\left(0,-\frac{2}{3}\right)\in \operatorname{int} D}$

AB $L(x,y,\lambda) = x^2+y^2-(x-y)^3+\lambda(x^2+y^2-2)$

$L_x = 2x-3(x-y)^2+2x\lambda$

$L_y = 2y+3(x-y)^2+2y\lambda$

$L_\lambda = x^2+y^2-2$

$$\left.\begin{array}{l} 2x-3(x-y)^2+2x\lambda=0 \\ 2y+3(x-y)^2+2y\lambda=0 \end{array}\right] +$$

$$2x+2y+2x\lambda+2y\lambda=0$$

$$2(x+y)+2\lambda(x+y)=0 \;/:2$$

$$(x+y)(1+\lambda)=0$$

$$\underline{x=-y} \quad \vee \quad \lambda=-1$$

$y^2+y^2-2=0$

$2y^2-2=0 \;/:2$

$y^2-1=0$

$(y-1)(y+1)=0$

$y=1 \;\vee\; y=-1$

$x=-1 \quad\;\; x=1$

$\boxed{M_5(-1,1)}$ $\boxed{M_6(1,-1)}$

$-3(x-y)^2=3(x-y)^2$

$6(x-y)^2=0$

$\underline{x=y}$

$y^2+y^2-2=0$

$2y^2-2=0$

$(y-1)(y+1)=0$

$y=1 \;\vee\; y=-1$

$x=1 \quad\;\; x=-1$

$\boxed{M_7(1,1)}$ $\boxed{M_8(-1,-1)}$

$M_5(-1,1) + x<0 \;\notin \operatorname{int} D$

$\boxed{M_6(1,-1)\in \operatorname{int} D}$

$\boxed{M_7(1,1)\in \operatorname{int} D}$

$M_8(-1,-1) + x<0 \;\notin \operatorname{int} D$

$\boxed{A(0,\sqrt{2})\in \operatorname{int} D}$

$\boxed{B(0,-\sqrt{2})\in \operatorname{int} D}$

$\frac{1}{6})$

$Z(M_2) = \frac{1}{36}+\frac{1}{36}-\frac{2^3}{36^3} = \frac{1}{18}-\frac{1}{18^3} = \frac{18^2-1}{18^3} = \frac{(18-1)(18+1)}{18^3} = \frac{17\cdot 19}{18^3} < 1$

$Z(M_3) = 0$

$Z(M_4) = \frac{4}{9}-\frac{8}{27} = \frac{12-8}{27} = \frac{4}{27}$

$Z(M_6) = 1+1-8 = 2-8 = -6$

$Z(M_7) = 1+1 = 2$

$Z(A) = 2+2\sqrt{2} = 2+2\cdot 1{,}41 = 2+2{,}82 = 4{,}82$

$Z(B) = 2-2\sqrt{2} = 2-2{,}82 = -0{,}82$

$$\boxed{\begin{array}{l} Z_{MIN} = Z(M_6) = -6 \\ Z_{MAX} = Z(A) = 4{,}82 \end{array}}$$

а) локални екстремуми

$$\left.\begin{aligned} z_x &= 2x-3(x-y)^2 \\ z_y &= 2y+3(x-y)^2 \end{aligned}\right\} \quad \begin{aligned} &M_1(0,0) \\ &M_2\left(\frac{1}{6},-\frac{1}{6}\right) \end{aligned}$$

$$\boxed{\begin{aligned} z_{xx} &= 2-6(x-y) \\ z_{yy} &= 2-6(x-y) \\ z_{xy} &= 6(x-y) \end{aligned}}$$

$\underline{M_1(0,0)}$

$\downarrow$

$r=2;\ t=2;\ s=0;\ \Delta=4$

$\Delta>0;\ r>0 \Rightarrow$ $\boxed{M_1\text{-е лок. мин.}}$

$\underline{M_2\left(\frac{1}{6},-\frac{1}{6}\right)}$

$\downarrow$

$r=2-\cancel{6}\cdot\frac{2}{\cancel{6}}=0$

$t=2-\cancel{6}\cdot\frac{2}{\cancel{6}}=0$

$s=\cancel{6}\cdot\frac{2}{\cancel{6}}=2$

$\Delta=-4$

$\Delta<0 \Rightarrow$ $\boxed{M_2\text{-е няма екстрем}}$

$\boxed{z_{мин} = z(M_1) = 0}$